Nonlinear dynamics of a fluid-filled hollow microcantilever subjected to flowing particles

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Summary. Nonlinear dynamics of a microfluidic cantilever interacting with a flowing buoyant particle inside the cantilever channel is investigated. With a U-shape and a non-uniform cross-section, the microbeam is internally filled with fluid and operates in an external air environment. An accurate reduced-order model of the system is obtained retaining geometric and inertia nonlinearities. It is found that the transient motion of the particle modifies the time history and the frequency response curve of the fluidic device, leading to jumps between coexisting stable attractors that could be exploited for mass sensing applications.

Introduction

Nano/Micro-Electro-Mechanical-Systems (NEMS/MEMS) have systematically improved as biosensor devices with the aim of mass detection of nanoparticles and biomolecules [1]. In particular, the capabilities of resonant cantilevers have attracted the interest of the community despite their simple structure, they have unprecedented sensitivity (single molecule) and are approaching as an alternative to the conventional mass spectrometry [2].
The detection schemes used by nano/microcantilever mass sensors rely on tracking the frequency shifts induced by the additional mass of the specimen. The operational interaction of the mass with the mechanical device can be through shallow techniques such as binding and surface adsorption. However, Suspended Microchannel Resonators (SMR) [3], based on hollow cantilevers, allow to place the specimen within the device. Thus, biological samples that require an aqueous environment can be analysed operating the cantilever in an non-acqueous surrounding medium increasing the sensitivity by orders of magnitude.
Mass sensing using cantilevers is widely described in literature, however hollow microcantilevers and their behaviour under the interaction of small particles have not been investigated in detail. Here, we present a numerical model to analyse linear and nonlinear dynamics of a hollow microcantilever subjected to a flowing particle.

Problem definition

The hollow microcantilever beam analysed is U-shaped with two legs as shown in Figure $1(a)$. The device is similar to the device reported by Ghatkesar et al [4], which was used as a micro-pipette to manipulate fluid with femto-litre volume precision. Here, we consider the device as a simple hollow channel without a hollow tip. The particle with mass M enters the channel from one end and exists from the other end. It moves with the fluid flow. The particle relative motion with respect to the fluid flow is neglected. The device is considered to be actuated by a piezoelectric base excitation.

Mathematical modelling

The governing equations describing the system dynamics, obtained by means of an energy approach, present distinctive features: as a consequence of the non-uniform shape (see Figure $1(a)$), the geometric properties are only stepwise constant. Moreover, the mass density depends on both the internal fluid filling the channel and the external environment. Since the beam is assumed to be operating in air, an additional inertial force is introduced to account for the movement of fluid particles sticking to the beam surface.
The model describing transverse displacement w of the beam is as following:

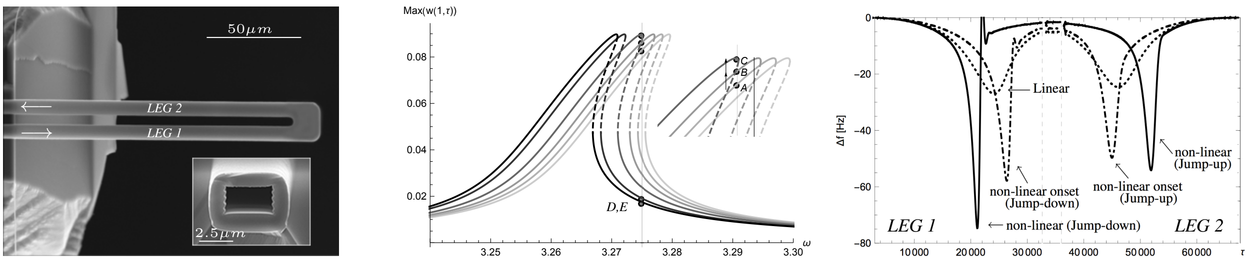
$$m\ddot{w} + E\left(Iw''\right)'' + \beta m_f w'' + E\left(Iw'w''^2\right)' + E\left(w'^2\left(Iw''\right)'\right)' +$$
$$\left[w'\int_L^r m\left(\int_0^r w'\ddot{w}' + \dot{w}'^2 dr\right)dr\right]' + c\dot{w} + M\left[V^2 w'' + 2V\dot{w}' + \ddot{w}\right]_{r=X}\delta\left(r-X\right) = G\cos\left(\omega t\right). \quad (1)$$

The present model accounts for the stiffness variation due to the internal fluid (β) that interacts with the inner walls of the beam [5]. Moreover, both geometric (forth and fifth term) and inertia (sixth term) nonlinearities are considered. The last term on the left-hand side of Eq.(1) is the time dependent effect due to the particle interaction that shall be evaluated at the specific position of the particle.

Results

Numerical simulations have been conducted by i) a pseudo arc-length continuation and collocation scheme to perform bifurcation analysis and obtain frequency response curves, ii) direct time integration of the equations of motion with a multistep Adams-Moulton method in order to obtain time histories and analyze the transient cantilever-particle interaction.
The moving particle inside the microchannel modifies the steady state response of the cantilever. By exciting the cantilever close and below its fundamental frequency, the motion is primarily driven by the first mode, and with the movement of the

added mass from the clamped edge to the free end, the oscillation amplitude increases. This behaviour is not persistent over the full travel of the particle. Indeed the added mass modifies the frequency response of the cantilever shifting the resonant frequency towards lower values as shown in Figure $1(b)$. This means that as long as the peak amplitude of the frequency response curve is above the amplitude of the excitation frequency, the particle runs on the left branch. Therefore, the amplitude of the motion continues to grow monotonically (moving from point A-C in Figure $1(b)$). Once the particle reaches the peak amplitude then the amplitude decreases abruptly as a consequence of passing a saddle-node bifurcation point. Here it should be noted that the response is largely affected by the excitation point and the combined effect of amplitude growth/reduction is found only if the cantilever is driven by an offset below the resonant frequency. In fact, an excitation beyond the resonant frequency leads to a seamless reduction of the amplitude response.



(a) (b) (c)

Figure 1: a) U-shaped cantilever. The inset image shows the cross-section of the legs. b) Change in frequency response induced by the particle. Points A, B, C depicts motion along the upper branch whereas points D, E belongs to the lower branch solution. c) Frequency drop in the time for the full particle travel. The dashed vertical lines represent the start and end of the motion in the free-end of the beam.

The inertial effect due to the particle movement also reduces the frequency of the cantilever in time. Again, a sudden frequency-drop associated with the transition between large (before the bifurcation point) and small amplitude motion (after the jump) in the nonlinear regime is found, and reported in Figure $1(c)$. Thus, if the aim is to exploit the time response or the frequency drop to perform mass sensing, the correct choise of both frequency and amplitude of the excitation will result in an optimal resolution [6]. Finally, by considering a full travel for the particle, from the inlet to the free-end and from the free-end to the outlet, an asymmetric response is observed that is more evident in the nonlinear case: a jump-down occurs in the motion within the firstleg whereas a jump-up is encountered in the outgoing travel of the buoyant mass.

Conclusions

In this work, nonlinear dynamics response of a fluid-filled hollow microcantilever has been investigated by accounting for the interaction with a flowing particle. A new model describing the hollow cantilever beam is presented. Numerical simulations have been performed to analyse the transient behaviour due to the beam-particle interaction. Detection of time-response peaks and frequency drops are evaluated in different regimes. It is observed that better resolutions can be achieved by exploiting the nonlinear response. The rich dynamics of the microbeam induced by the moving particle can potentially be applied to perform accurate mass sensing without using frequency tracking hardware.

References

[1] Ilic, B. et al. (2001) Single cell detection with micromechanical oscillators *Journal of Vacuum Science & Technology B* **19**: 2825-2828

[2] Hanay, M. Selim, et al. (2015) Inertial imaging with nanomechanical systems.*Nature nanotechnology* **10**(4): 339-344

[3] Burg, T. et al. (2007) Weighing of biomolecules, single cells and single nanoparticles in fluid *Nature* **446**(7139): 1066-1069

[4] Ghatkesar, M.K. et al. (2014) Hollow AFM cantilever pipette *Microelectronic Engineering* **124**: 22-25

[5] Zhang W.M. (2016) Dynamics of suspended microchannel resonators conveying opposite internal fluid flow: Stability, frequency shift and energy dissipation *J. Sound and Vibration* **368**:103-120

[6] Nguyen, V.-N. et al. (2015) Bifurcation-based micro-/nanoelectromechanical mass detection *Nonlinear Dynamics* **79**(1):647-662