Stability Analysis of Machining Processes with Parameter Uncertainty

Dominik Hamann, Nico-Philipp Walz, Achim Fischer, Michael Hanss, Peter Eberhard
Institute of Engineering and Computational Mechanics, University of Stuttgart, Germany

Summary. Uncertainty analysis of stability lobe diagrams is an essential step for the selection of efficient and solid operating conditions. The definition of uncertain parameters can enable the illustration of the effect of required model simplifications or the vagueness of parameter definitions. Additionally, a deeper insight into the system behavior is possible. The additional analysis has an high contribution to the computation effort, thus, an efficient implementation is necessary, which concurrently ensures reliability of the solution scheme.

Introduction

Machining processes are key manufacturing steps in almost every modern industry. On the one hand high material removal rates are sought to decrease cycle times and, thus, reduce the expense. On the other hand tool wear and surface quality are important criteria as well. Here, stability analysis of the machining processes plays an important role to find parameters for efficient and save machining processes. The reliability of the prediction based on stability analysis is significantly dependent on the reliability of the models, chosen parameters, and the stability analysis algorithm itself.
Simplifications in the model or vagueness of the definition of parameters are epistemic uncertainties. In contrast, aleatory uncertainties stem from random effects. Thus, we take a possibilistic approach, fuzzy arithmetic, instead of common probabilistic approaches. It is the focus of the uncertainty analysis to detect operating conditions considering the possibility of (in-)stability due to the defined uncertainty.
For the uncertainty analysis of the stability lobe diagrams, the milling process with a helical tool are presented with focus on helix-induced flip islands, [1]. The stability limit of the nominal system exhibit neither a closed region nor convexity, which are interesting characteristics for the uncertainty analysis.

Helical Milling Model

The milling process with a helical tool is modeled as

$$\ddot{\xi}(t) + 2\zeta\omega_{\mathrm{n}}\dot{\xi}(t) + \omega_{\mathrm{n}}\xi(t) = -G(t, a_{\mathrm{p}})(\xi(t) - \xi(t-\tau)) \tag{1}$$

with

$$G(t, a_{\mathrm{p}}) = \sum_{j=1}^{N} \frac{q f_{\mathrm{z}}^{q-1}}{m} \frac{N l_{\mathrm{p}}}{2\pi} \int_{\varphi_{\mathrm{en},j}}^{\varphi_{\mathrm{ex},j}} \sin^q \varphi (k_{\mathrm{t}} \cos\varphi + k_{\mathrm{r}} \sin\varphi) \mathrm{d}\varphi \, , \tag{2}$$

see [1]. The delay differential equation considers the surface regeneration of the workpiece, which can lead to instabilities, called chatter. The model exhibit an interesting stability lobe diagram.

Stability Analysis

Modeling machining processes by time delayed differential equations and their stability analysis is a difficult task. Both have received and still receive a lot of interest in the scientific community. The first-order system reads $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{x}(t-\tau)$ with $\mathbf{A}(t+T) = \mathbf{A}(t)$, $\mathbf{B}(t+T) = \mathbf{B}(t)$ and $T = \tau$. Discretization of a period T and mapping yields the approximated system

$$\mathbf{y}_{i+1} = \mathbf{\Phi}\mathbf{y}_i \, . \tag{3}$$

The stability of the system can be determined by the magnitude of the critical eigenvalue λ_1 of the approximated monodromy matrix $\mathbf{\Phi}$. The system is stable if all eigenvalues are located inside the unit circle of the complex plane, i.e. $|\lambda_1| \leq 1$.

Uncertainty Analysis

Epistemic uncertainties express a vagueness in the definition of parameters. Triangular fuzzy numbers, denoted as $\tilde{p} = \mathrm{tfn}(\bar{p}, \alpha_l, \alpha_r)$, are most widely used as they require minimal information and assumptions. The nominal value is denoted by $\bar{p}$ and the left and right hand worst-case deviation by α_l and α_r. The membership function $\mu_{\tilde{p}}(y)$ takes values between 0 and 1.
Considering the mapping $f : \mathcal{Y} \mapsto \mathcal{Z}$, with the domain of uncertain parameters $\mathcal{Y} \subseteq \mathbb{R}^n$ and the output of the function $\mathcal{Z} \subseteq \mathbb{R}$, Zadeh's extension principle defines the fuzzy result $\tilde{v} = f(\tilde{\mathbf{p}})$ by

$$\mu_{\tilde{v}}(z) = \begin{cases} \sup_{\mathbf{y} \in \mathcal{Y}_z} \mu_{\tilde{\mathbf{p}}}(\mathbf{y}) & \text{for } \mathcal{Y}_z \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and $\mathcal{Y}_z = \{\mathbf{y} \in \mathcal{Y} \mid f(\mathbf{y}) = z\}$. The multivariate membership function of independent parameters reads $\mu_{\tilde{\mathbf{p}}}(\mathbf{y}) = \min(\mu_{\tilde{p}_i}(y_i))_{i=1}^{n}$.
For the stability analysis, the function may be formulated as $g : \mathcal{X} \times \mathcal{Y} \mapsto \mathcal{Z}$ with the domain of operating conditions $\mathcal{X} \subset \mathbb{R}^2$, i.e. spindle speed Ω and axial immersion a_p. Using the critical eigenvalue, the function reads $g(\mathbf{x}, \mathbf{y}) = |\lambda_1(\mathbf{x}, \mathbf{y})|$. Furthermore, the stability limit is implicitly formulated, corresponding to $g(\mathbf{x}, \mathbf{y}) = 1$. The adaption of the extension principle for this problem is

$$\mathcal{Y}_\mathbf{x} = \{\mathbf{y} \in \mathcal{Y} \mid g(\mathbf{x}, \mathbf{y}) = 1\} \,. \tag{5}$$

There are different solution schemes available for the calculation of the fuzzy stability limit. The application of the transformation method [2] has been presented in [4]. Further solution schemes are presented in [3].

Investigations

The calculation of stability lobe diagrams is connected to high computational effort. Uncertainty analysis of the stability lobe diagrams yields a multiple of the computational effort, thus, efficient solution schemes are sought. Possible solutions schemes can base on different sampling strategies or optimization. Although, the solution schemes should be adapted to the characteristics of the function of the critical eigenvalue $g(\mathbf{x}, \mathbf{y})$ as good as possible. These characteristics can be stated as continuous but not continuously differentiable, and non-monotonic, see Fig. 1

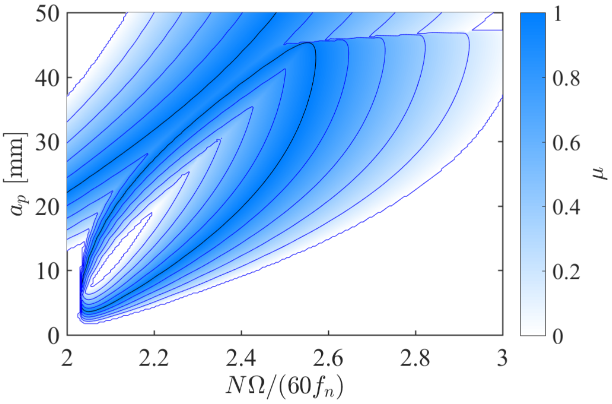

Figure 1: Fuzzy stability lobe diagram of the helical milling model with uncertain cutting-force exponent $\tilde{q} = \mathrm{tfn}(0.75, 0.075, 0.075)$

Discretization by a full grid takes about 2^{n-1} as many evaluations as by the transformation method, note n is the number of uncertain parameters. The transformation method is an excellent discretization scheme if the investigation is interested in the entire fuzzy output $\tilde{v}$. For stability analysis the investigation is focused on the stability limit, thus, the aim of the calculation scheme should be the calculation of the stability limit $g(\mathbf{x}, \mathbf{y}) = 1$. Using bisection methods, as [5], the calculation effort can possibly be reduced even further.
Optimization approaches are unfavorable solution schemes for this application, since the domain of the stability lobe diagram $\mathcal{X}$ has to be discretized first and optimization problems with nonlinear constraints, see (4) and (5), have to be solved individually. However, optimization is a possible solution scheme as well.

Conclusions

Stability lobe diagrams are important for the evaluation of appropriate machining processes or operating conditions as spindle speed or axial immersion. The reliability of these diagrams is fundamental. Hence, uncertainties in the modeling due to conscious simplifications or by the reason of lack of knowledge, or the vagueness of parameter definition, have to be considered. The implementation can be obtained by using fuzzy numbers in the modeling step. An efficient and solid solution scheme for the formulated uncertainty analysis is considered.

References

[1] Insperger, T.; Stepan, G.: Semi-Discretization for Time-Delay Systems: Stability and Engineering Application. New York: Springer, 2011

[2] Hanss, M.: Applied Fuzzy Arithmetic - An Introduction with Engineering Applications. Heidelberg: Springer, 2005

[3] Walz, N.P.: Fuzzy Arithmetical Methods for Possiblistic Uncertainty Analysis. Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart, Nr. 47. Aachen: Shaker, 2016.

[4] Sims, N.D.; Manson, G.; Mann, B.P.: Fuzzy Stability Analysis of Regenerative Chatter in Milling. Journal of Sound and Vibration, Vol. 329, No. 8, pp. 1025-1041, 2009.

[5] Bachrathy, D.; Robust Stability Limit of Delay Dynamical Systems. Periodica Polytechnica. Engineering. Mechanical Engineering. Vol. 59, No. 2, pp. 74-80 2015.