Pulses and snakes in the Ginzburg–Landau equation

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Summary. Using a variational formulation for partial differential equations combined with numerical simulations on ordinary differential equations (ODEs), we find two categories (pulses and snakes) of dissipative solitons, and analyse the dependence of both their shape and stability on the physical parameters of the cubic-quintic Ginzburg–Landau equation (CGLE). In contrast to the regular solitary waves investigated in numerous integrable and non-integrable systems over the last three decades, these dissipative solitons are not stationary in time. Rather, they are spatially confined pulse-type structures whose envelopes exhibit complicated temporal dynamics. Numerical simulations reveal very interesting bifurcations sequences as the parameters of the CGLE are varied. Our predictions on the variation of the soliton amplitude, width, position, speed and phase of the solutions using the variational formulation agree with simulation results. Firstly, we develop a variational formalism which explores the various classes of dissipative solitons. Given the complex dynamics, the trial functions have been generalized considerably over conventional ones to keep the shape relatively simple, and the trial function integrable while allowing arbitrary temporal variation of the amplitude, width, position, speed and phase of the pulses and snakes. In addition, the resulting Euler–Lagrange (EL) equations from the variational formulation are treated in a completely novel way. Rather than considering the stable fixed points which correspond to the well-known stationary solitons, we use dynamical systems theory to focus on more complex attractors, viz. periodic (pulses) and quasiperiodic (snakes). Periodic evolution of the trial function parameters on stable periodic attractors yields solitons whose amplitudes and widths are non-stationary or time dependent. Secondly, we investigate the dissipative solitons of the CGLE and analyse its qualitative behaviour by using numerical methods for ODEs. To solve numerically the nonlinear systems of ODEs that represent EL equations obtained from variational technique, we use an explicit Runge–Kutta fourth-order method. Finally, we elucidate the Hopf bifurcation mechanism responsible for the various pulsating solitary waves, as well as its absence in Hamiltonian and integrable systems where such structures are absent due to the lack of dissipation.

Keywords

Integrable systems, Ginzburg–Landau, Solitons, Variational formulation, Pulses, Snakes