Dynamic release condition for latched curved micro beams

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Summary. Curved beams subjected to transverse force may exhibit latching phenomena, namely remain in their buckled configuration under zero force. By satisfying the conditions which grant latching, it is possible to shift a bistable beam to its second stable state, thereby holding ("trapping") it under zero load. This procedure is applicable for beams which their second stable branch is either accessible or inaccessible under quasi-static loading. However, in order to induce a snap-back reaction in latched trapped beams, an actuation at an opposite force is required. As a result, two independent electrodes are needed for actuation under electrostatically load. In this study, we present that a snap-back can be induced dynamically, reducing the two electrode system to one. The analysis is based on a reduced order (RO) model resulting from the Galerkin decomposition with buckling modes of a straight beam as base functions. The result of the said analysis grants a necessary condition, which the beam must apply in order to induce a dynamic release response.

Introduction

For a certain range of geometrical parameters, a curved beam may exhibit three limit points, i.e. snap-through (S), release (R) and pull-in (PI), making the beam bistable [1]. In [2] it was found, that a bistable beam can have a latching point, which resides at zero load. Such a configuration presents a singular scenario, where a beam can snap-through to its second stable equilibrium and remain there, unable to return to its initial position under quasi-static load. Such a scenario dictates, that in order to establish a two way switching, a two electrode system needs to be designed. However, as was shown, it is possible to release the beam by pre-loading it to a certain location, causing it to gain strain energy in the process. By releasing the beam from its new location, the beam can be released dynamically to its initial equilibrium. In this study, we take the said observation and analyze the dynamic release response. It is shown, that depending on the damping and initial condition (the "new" location on the second stable branch), the beam can either converge to its initial equilibrium or back to its latching point [3].

Model

The geometry of the beam is characterized by its initial elevation $\hat{h}_0$ (defined as the distance between the mid-point of the beam and the line connecting the clamped ends), thickness $\hat{d}$, width b, and length L. The beam resides at a distance g_0 (also known as the gap) from an actuating electrode, providing a distributed electrostatic load, as seen in Fig. 1(a). Assuming a symmetric beam response, we employ a single degree of freedom (DOF) reduced order (RO) model, obtained using the Galerkin decomposition, to describe the beam dynamics. The non-dimensional equation of motion is given by

$$\ddot{q} + 2\xi\omega_n\dot{q} + \frac{4\pi^4}{3}\left(4\left(q-h_0\right) + \frac{6}{d^2}\left(q^2-h_0^2\right)q\right) + \frac{4\beta\left(t\right)}{3\sqrt{\left(1+q\right)^3}} = 0 \tag{1}$$

with initial conditions $q_0 = h_0$, $\dot{q_0} = 0$. h_0, d and β are the beam non-dimensional initial elevation, thickness and voltage parameter, respectively, normalized against the gap. ξ and ω_n are the linearized non-dimensional damping ratio and natural frequency, respectively. Note, that for a single mode representation, the normalized location of the beam is bounded by $-1 \leq q \leq h_0$ (see [3] for the full development).

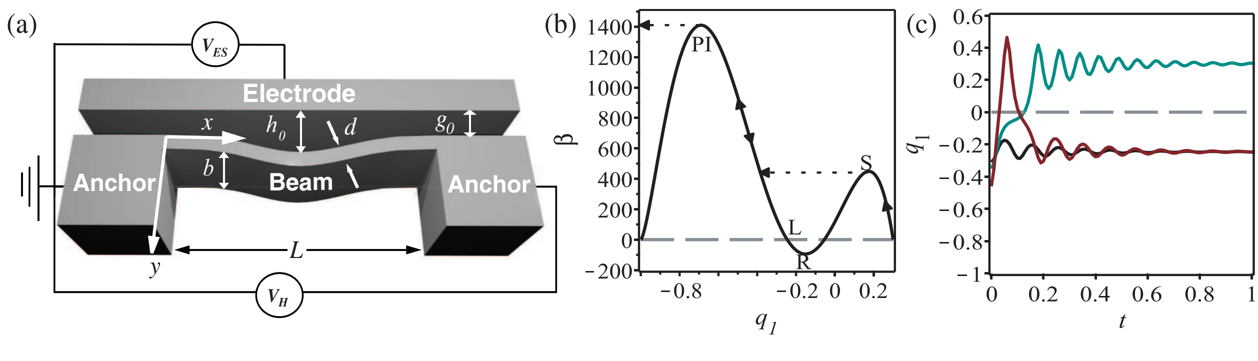

Figure 1: (a) Illustration of an initially curved beam under electrostatic actuation. (b) Predicted static equilibrium, solved from the static counterpart of Eq. (1)) for a beam with non dimensional thickness and initial elevation of $d = 0.1$ and $h = 0.3$, respectively. Arrows represent beam movement on the equilibrium path and response upon reaching the snap-through and pull-in voltages, presented as β_S and β_{PI}, respectively. Since the beam has a negative release voltage, i.e. $\beta_R < 0$, a latching point (L) exists. Dashed grey line represents $\beta = 0$ (c) Calculated responses of the beam from different initial locations, assuming a quality factor of $Q = 10$, with $q_0 = -0.31$, $q_0 = -0.35$ and $q_0 = -0.31$ in black, green and red, respectively. Dashed grey line represents $q = 0$.

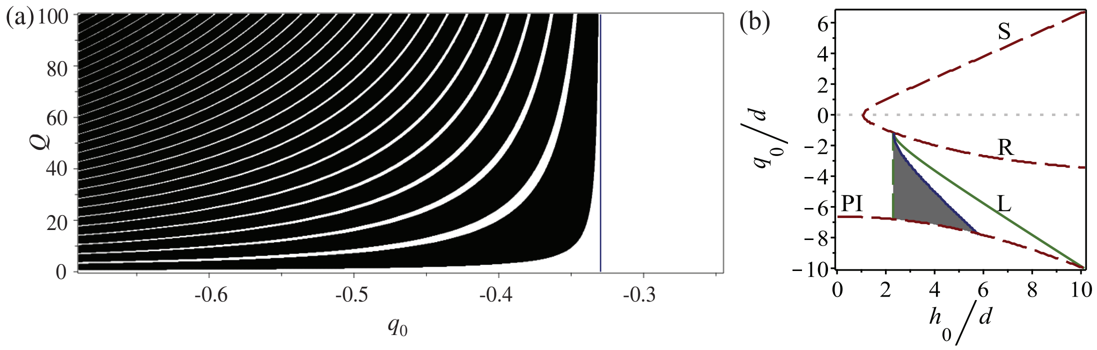

Figure 2: (a) Actuation map pertaining to the configuration presented in Fig. 1(b), signifying the two different possible outcomes depending on the existing damping (Q), and the the initial condition given to the beam. The white region to the right of the necessary condition (the blue line) represents the conditions for which the beam returns to its latching point; the black regions represents the conditions for which the beam is dynamically released to its initial stable state. The white regions to the left of the condition represent bouncing. The map is bounded between the latching point $q_L = -0.2457$ and pull-in point $q_{PI} = -0.6898$. (b) Limit point map in dashed red lines, showing the pull-in (PI) and release (R) locations for $d = 0.1$, superimposed against the latching (L) location in solid green. The blue line represent the necessary condition for dynamic release (DR), stating that every point in the gray area beneath it satisfies the condition.

Dynamic release

A beam which adheres to both bistability and latching conditions is given in Fig. 1(b), showing all three limit points (S, R, PI) and a latching point (L), caused by the presence of a negative release voltage. To ascertain the different responses inherent in the beam, Eq. (1) was taken with $\beta = 0$ for a specific damping (stated by the Q-factor) and various initial locations q_0. In doing so, we assume that the beam has reached a specific location on its equilibrium curve by slowly preloading it. The calculation effectively begins when the actuated voltage drops to zero. Fig. 1(c) describes three possible scenarios, caused from preloading to three different locations, showing two possible outcomes. The beam can either converge back to q_L (the black response), converge to its initial location h_0 (the green response), or preform a trajectory around the initial location, only to return to q_L (i.e. “bouncing” as the red response). At this point, it is of importance to observe, that by taking the load to zero, the results are relevant for all loading types. However, in the presence of a nonlinear loading such as electrostatic load, a single difference is present. The difference lies in the limitation of the initial location q_0, presented by the pull-in point q_{PL}, bounding the range of possible initial locations, defined between the latching and the pull-in points, i.e. $q_{PI} \leq q_0 \leq q_L$.

Necessary condition

To see the overall behavior of the phenomenon, Eq. (1) was solved again for $\beta = 0$, by including the entire range possible under electrostatic load. In addition, the effect of the surrounding damping was taken into account for each initial location, thereby creating a map showing the two distinct possibilities. The calculations were done using the Runge-Kutta Fehlberg method in steps of $\Delta Q = 0.25$ and $\Delta q_0 = -0.00025$ for the damping and the initial location respectively, while the initial velocity was determined as $\dot{q}_0 = 0$. The result is given in Fig. 2(a) showing that a dynamic release is not guaranteed, and depends heavily on the damping, as was presented for the case of dynamic trapping [3]. Closer observation in Fig. 2(a) revels that a boundary, marking where a release can potentially occur. By taking Eq. (1), substituting $\beta = 0$ & $\xi = 0$, and by demanding that its phase plane response trajectory will cross $q = 0$ twice, a necessary condition can be extracted. Such a condition is granted in in Fig. 2(a) for $h_0 = 0.3$ and $d = 0.1$, completing the map and in Fig. 2(b) for $d = 0.1$, stating that the area beneath the black line guaranties that such a trajectory takes place. However, since the condition does not take into account the effect of damping, the condition is only a necessary one. With respect to electrostatic load, since the condition is upper bounded by q_{PL}, the area beneath the condition is limited by the pull-in line, thereby creating a closed area.

Conclusions

To conclude, a dynamic release of a latched curved beam from its latching point back to its initial location is possible, granted that the correct damping is present. The reason for the phenomenon lies in the dissipation rate of the strain energy, accumulated during the preloading of the beam, thereby creating two different outcomes. However, formulation of a necessary condition was carried out, showing that under the correct geometric parameters and initial locations, a dynamic release can potentially ensue.

References

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