Period-1 oscillations of a state-dependent delayed TCP model with PIE queue management policy via High-dimensional Harmonic Balance method

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Summary. PIE queue policy is an important method of controlling queuing routers delays in real internet. So it is valuable to investigate the state-dependent delayed fluid model for compound TCP coupled with PIE. But unfortunately the mathematical theory of SD-DDEs, especially for those with implicit function of delay, was just recently developed. By HDHB method, we cast the dependent variables in the domain and stored at $2N+1$ equally spaced sub-time levels over the period of the limit cycle. SDD $\tau(t)$ also is discretized in time domain. Thus it avoids tedious derivations of algebraic expressions for the Fourier coefficients of the nonlinear terms of the dynamical system. Results obtained by HDHBM are validated by comparisons with those of WinPP and DDE-BIFTOOL. Period-1 oscillations of the TCP model with PIE are proven.

As there are large and unmanaged router buffers in the Internet, it is necessary for us to study the problem of excessive queuing delays (bufferbloat). The Proportional Integral Enhanced (PIE) queue policy has shown its advantage of explicitly controlling queuing delays in routers. In addition, it incurs very small overhead and can be simply implemented in both hardwares and softwares, as the design does not require per-packer extra processing. What's more, PIE can ensure low latency and achieve high link utilization under various congestion situation. Therefore, it is significant to investigate the fluid model for compound TCP coupled with the PIE queue policy. The Compound-PIE system with an exogenous, non-dimensional parameter κ is described by the following set of equations [1, 2],

$$\begin{aligned} \dot{w}(t) &= \kappa\big(i(w)(1-p(t-R(t))) - d(w)p(t-R(t))\big)\frac{w(t-R(t))}{R(t)}, \\ \dot{q}(t) &= \kappa\big(\frac{w(t)}{R(t)}(1-p(t)) - C\big), \\ \dot{p}(t) &= \kappa\big(\mu\frac{\tau(t)-\tau_{ref}}{T} + \frac{1}{C}(\xi+\frac{\mu}{2})\big(\frac{w(t)}{R(t)}(1-p(t))-C\big)\big), \end{aligned} \tag{1}$$

$$i(w) = \frac{\alpha w(t)^k}{w(t)},\ \ d(w) = \beta w(t),\ \ R(t) = \tau(t) + T_p,\ \ \tau(t) = \frac{q(t)}{C}, \tag{2}$$

where w is the average window size of the long-lived TCP flows, p denotes the packet loss probability, q is the queue length. Note that α, k and β are the protocol parameters, μ, ξ are tunable PIE parameters, and τ_{ref} is the target queuing delay, T is the update rate of the packet loss calculations. Assume that the service rate of the link is C units. κ and α are chosen as bifurcation parameters in our papers. The matrix form of the equations can be written as

$$\dot{\mathbf{X}}(\tilde{t}) = \mathbf{f}\big(\mathbf{X}(\tilde{t}), \mathbf{X}(\tilde{t}-\sigma)\big). \tag{3}$$

In this paper, we apply the High-Dimensional Harmonic Balance method (HDHBM) to this model, which is derived from the HB method [3]. But it avoids the tedious derivations of algebraic expressions for the Fourier coefficients of the nonlinear terms of the dynamical system as in the classical HB method. To the best of our knowledge, few works of HDHBM have been reported in the state-dependent delayed differential equations except for [4]. The first step of both methods is to normalize $\tilde{t}$ as $t = \omega\tilde{t}$, where $\omega = 2\pi/T$. Then the T-periodic solutions of the original equation are transformed to the 2π-periodic solutions of the new equation, which can be approximated by the truncated Fourier serise expansion of N harmonics with the unknown Fourier coefficient variables $\hat{Q}_x^{(i)}$. In the HB method, we should solve the obtained equation to calculate the Fourier coefficient variables directly after substituting $X(t)$ and $X(t-\tau)$ into the resulting equations. While in the HDHB method, the second step follows, i.e., we cast the dependent variables in the domain and stored at $2N+1$ equally spaced sub-time levels over the period of the limit cycle. We obtain the Fourier coefficients via the $2N+1$-dimensional Fourier transformation matrix [3]

$$F = \begin{pmatrix} 1 & sin(t_0) & cos(t_0) & \cdots & sin(Nt_0) & cos(Nt_0) \\ 1 & sin(t_1) & cos(t_1) & \cdots & sin(Nt_1) & cos(Nt_1) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & sin(t_{2N}) & cos(t_{2N}) & \cdots & sin(Nt_{2N}) & cos(Nt_{2N}) \end{pmatrix}, \tag{4}$$

thus

$$Q_x = F\hat{Q}_x, \tag{5}$$

and

$$Q_x = \begin{pmatrix} x_1(t_0) & x_2(t_0) & \cdots & x_n(t_0) \\ x_1(t_1) & x_2(t_1) & \cdots & x_n(t_1) \\ \vdots & \vdots & \vdots & \vdots \\ x_1(t_{2N}) & x_2(t_{2N}) & \cdots & x_n(t_{2N}) \end{pmatrix}. \tag{6}$$

Now the problem is transformed into solving the equation with the dependent variables at time $t_j = j(2\pi/(2N+1))$,

$$\omega \mathbf{J}\mathbf{F}^{-1}\mathbf{Q}_{\mathbf{x}} - \mathbf{F}^{-1}\mathbf{R}_{\mathbf{x}} = \mathbf{0}, \tag{7}$$

where

$$\mathbf{R}_{\mathbf{X}} = \begin{pmatrix} f_1(X(t_0), X(t_0-\tau)) & f_2(X(t_0), X(t_0-\tau)) & \cdots & f_n(X(t_0), X(t_0-\tau)) \\ f_1(X(t_1), X(t_1-\tau)) & f_2(X(t_1), X(t_1-\tau)) & \cdots & f_n(X(t_1), X(t_1-\tau)) \\ \vdots & \vdots & \vdots & \vdots \\ f_1(X(t_{2N}), X(t_{2N}-\tau)) & f_2(X(t_{2N}), X(t_{2N}-\tau)) & \cdots & f_n(X(t_{2N}), X(t_{2N}-\tau)) \end{pmatrix}. \tag{8}$$

The SDD $\tau(t)$ also is discretized in the time domain since it is intrinsically not constant. Multiplying the left of both sides of equation (7) by $\mathbf{F}$, we obtain

$$\omega \mathbf{F}\mathbf{J}\mathbf{F}^{-1}\mathbf{Q}_{\mathbf{x}} - \mathbf{R}_{\mathbf{x}} = \mathbf{0}. \tag{9}$$

The new equation does not require symbolic calculations as in the classical HB method, avoiding the tedious computations, which makes it accessible for an existing time-marching code.

We capture successfully the limit cycles of period-1 oscillations in the model (1) by using the HDHBM. Moreover, in this paper the first harmonic amplitudes versus the bifurcation parameters κ, α are respectively demonstrated by the HDHBM with three harmonics (HDHBM3). The results displayed in Figs. 1 and 2 excellently agree well with the data obtained by the nonlinear dynamical software WinPP and the Matlab-based DDE bifurcation analysis package, DDE-BIFTOOL [5]. It discloses the existence of the period-1 oscillations in the state-dependent delayed TCP model with a Proportional Integral Enhanced (PIE) queue management policy. It also indicates the effects of the state-dependent delay on the dynamics of the TCP model. Thus it also implies the effectiveness and validity of HDHBM in state-dependent delayed differential equations.

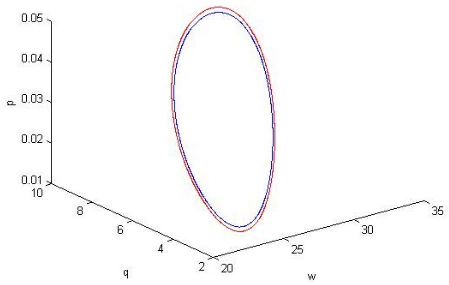

Figure 1: The comparison of phase portraits of period-1 oscillation of system (1) by HDHBM and WinPP with $\kappa = 1,\ \alpha = 0.865,\ \beta = 0.5,\ C = 100,\ Tp = 0.2,\ k = 0.75,\ \mu = 0.01,\ \xi = 0.5,\ \tau_{ref} = 0.0605,$ and $T = 0.03$. Red and black lines are respectively the results of WinPP and HDHBM3.

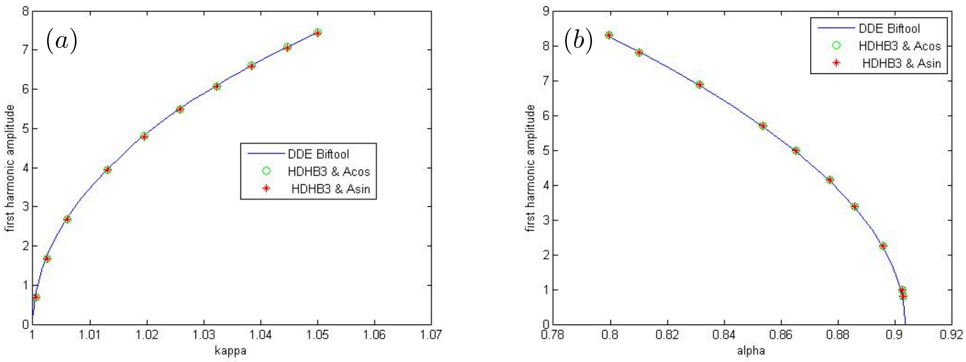

Figure 2: Comparisons of first harmonic amplitudes of w respectively versus the bifurcation parameters κ (in (a)) and α (in (b)) by HDHBM3 and DDE-BIFTOOL, where $(a)\ \alpha = 0.9936$ and $(b)\ \kappa = 1$, with the same set of parameters $\beta = 0.5,\ C = 100,\ Tp = 0.2,\ k = 0.75,\ \mu = 0.01,\ \xi = 0.5,\ \tau_{ref} = 0.0651,$ and $T = 0.03$. Solid line: DDE-BIFTOOL; open circles: HDHBM3 for the case of the vanishing sin(t); stars: HDHBM3 for the case of the vanishing cos(t).

References

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