**Competitive Modes as Reliable Predictors of Chaos versus**

**Hyperchaos and as Geometric Mappings Accurately Delimiting**

**Attractors**

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**Abstract**

We consider real quadratic dynamics in the context of competitive modes, which allows us to view chaotic systems as ensembles of competing nonlinear oscillators. We find that the standard competitive mode conditions may in fact be interpreted and employed slightly more generally than has usually been done in recent investigations, with negative values of the squared mode frequencies in fact being admissible in chaotic regimes, provided that the competition among them persists. This is somewhat reminiscent of, but of course not directly correlated to, ‘stretching (along unstable manifolds) and folding (due to local volume dissipation)’ on chaotic attractors. This new feature allows for the system variables to grow exponentially during time intervals when mode frequencies are imaginary and comparable, while oscillating at instants when the frequencies are real and locked in or entrained. In addition to an application of the method to chaotic attractors, we consider systems exhibiting hyperchaos, and conclude that the latter exhibit three competitive modes rather thantwo for the former. Finally, in a novel twist, we re-interpret the components of the Competitive Modes analysis as simple geometric criteria to map out the spatial location and extent, as well as the rough general shape, of the system attractor for any parameter sets corresponding to chaos. The accuracy of this mapping adds further evidence to the growing body of recent work on the correctness and usefulness of competitive modes. In fact, it may be considered a strong ’a posteriori’ validation of the

Competitive Modes conjectures and analysis.

**Keywords**: quadratic dynamics; hyperchaos; chaos diagnostics; nonlinear oscillators