Nonlinear Dynamics Analysis of a Rotor-Damper System Through Nonlinear Galerkin Method on Approximate Inertial Manifold

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Summary. The nonlinear dynamical response is presented for a rotor supported by squeeze film dampers and ball bearings. The nonlinear Galerkin method associated with approximate inertial manifold is employed to reduce the degree-of-freedoms of the original system for an efficient implementation of nonlinear vibration analysis. With the verified reduced model, the steady response of the rotor is solved considering mass unbalance, nonlinear damping and bearing forces. The existence of multiple frequencies factional to the rotation speed is presented, and the bifurcations in motion are identified with respect to the rotation speed. Based on numerical results, chaotic motion can be developed through period doubling and intermittency for the rotor system furnished with nonlinear squeeze film damper and ball bearings.

Background of the Research

Rotor assemblies are the core components of aircraft engines and gas turbines. The dynamical behaviour of a rotor system is complicated owing to not only the gyroscopic effect but also the presence of bearings and dampers. Particularly, squeeze-film dampers are nowadays mounted on rotor assemblies to provide strong resisting force against external shock loads. The damper force, in essence, is as a highly nonlinear function of displacement and velocity of the disturbed rotor. For such a nonlinear system, the efficiency of motion analysis is an important issue. Various methods can be used to reduce the order of the original system to a more compact one for more efficient nonlinear analysis. It is known that, for each dynamical system, there exists an invariant global attractor on inertial manifolds that restricts the dynamics of the original system and reduces it to a lower-dimensional subsystem. However, it is hard to isolate such an inertial manifold from the original system. Instead, an approximate inertial manifold proposed by Foias et al. [1] can be introduced to determine the long term behaviour of dissipative dynamic systems. Based on this concept, a lower order subsystem can be constructed through approximating the slave subsystem as a function of the master subsystem [2,3], and the reduced master subsystem can subsequently be solved numerically as long as its order is much smaller than the original system for the purpose of computational efficiency.
In this paper, the nonlinear dynamical response is presented for a rotor supported by squeeze film dampers and ball bearings. The nonlinear Galerkin method associated with approximate inertial manifold is employed to obtain a reduced dynamical model for a quick implementation of nonlinear vibration analysis. The existence of multiple frequencies factional to the rotation speed is presented, and the bifurcations in motion are identified. It is found that chaotic motion can be developed through the route of period doubling and intermittency.

Mathematical Formulation, Method and Result

Equation of motion of the rotor system

Considering the effects of rotatory inertia and gyroscopic force, the equation of motion for a rotor can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\left(\mathbf{C}+\mathbf{G}\right)\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t) \tag{1}$$

where $\mathbf{q}(t)$ is displacement of the shaft center. Herein, only the lateral motion of the rotor is considered. $\mathbf{M}$, $\mathbf{C}$, $\mathbf{G}$ and $\mathbf{K}$ are matrices of mass, damping, gyroscopic and structural stiffness. $\mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t)$ represents the resultant forces due to mass unbalance, ball bearings and squeeze-film dampers (SFDs). Particularly, the reactions of the SFDs in the radial and tangential directions can be expressed as a set of nonlinear function of displacement and velocity, as follows

$$F_r=-\mu RL^3c^{-2}\left(I_1\dot{\varepsilon}+I_2\varepsilon\dot{\phi}\right),F_t=-\mu RL^3c^{-2}\left(I_2\dot{\varepsilon}+I_3\varepsilon\dot{\phi}\right) \tag{2}$$

where R, c, L are nominal radius, radial clearance and length of the damper, respectively; μ is the dynamic viscosity of lubricant. ε is the non-dimensional displacement, and ϕ the attitude angle of the damper center. I_1, I_2 and I_3 are integration constants depending of ε and ϕ.

Model reduction with nonlinear Galerkin method

The nonlinear Galerkin method is employed to reduce the original dynamical model together with the construction of approximate inertial manifold. To this end, equation (1) is expressed as first-order ordinary differential equations

$$\dot{\mathbf{u}}(t)+\mathbf{g}(\mathbf{q},\dot{\mathbf{q}},t)=\dot{\mathbf{u}}+\mathbf{A}\mathbf{u}+\mathbf{h}(\mathbf{u},t)=0,\ \mathbf{u}=(\mathbf{q},\dot{\mathbf{q}})\in R^n \tag{3}$$

Adopting the Galerkin method, we expand $\mathbf{u}(t)$ in term of responses of master- and slave-subsystems

$$\mathbf{u}(t)=\mathbf{Y}_m\boldsymbol{\xi}(t)+\mathbf{Z}_p\boldsymbol{\eta}(t),\ \ m+p=n. \tag{4}$$

where $\mathbf{Y}_m\in R^{n\times m}$ and $\mathbf{Z}_p\in R^{n\times p}$ are partial modal matrices composed of the first m and the rest p=n-m eigenvectors of $\mathbf{A}$, respectively. Similarly, let the partial modal matrices of $\mathbf{A}^T$ be $\tilde{\mathbf{Y}}_m^T$ and $\tilde{\mathbf{Z}}_p^T$. Following Titi [3], the dynamics of the slave subsystem can be described on an approximate inertial manifold that is constructed in term of $\boldsymbol{\eta}$

$$\boldsymbol{\eta} = \Phi_{aim}(\boldsymbol{\xi}) = -\mathbf{J}^{-1}\tilde{\mathbf{Z}}_p^T \mathbf{g}(\mathbf{Y}_m\boldsymbol{\xi} + \mathbf{Z}_p\boldsymbol{\eta}, t) \tag{5}$$

where $\mathbf{J} = \tilde{\mathbf{Z}}_p^T \mathbf{A}\mathbf{Z}_p$. The master subsystem is now solved with a reduced number of degree of freedom, governed by

$$\dot{\boldsymbol{\xi}} + \tilde{\mathbf{Y}}_m^T \mathbf{A}\mathbf{Y}_m\boldsymbol{\xi} + \tilde{\mathbf{Y}}_m^T \mathbf{g}(\mathbf{Y}_m\boldsymbol{\xi} + \mathbf{Z}_p\Phi_{aim}(\boldsymbol{\xi}), t) = 0 \tag{6}$$

which is relatively easy to handle compared with the original system in Eq. (3). A fixed-point iteration method can be used to accelerate the convergence of $\boldsymbol{\eta}$. The details of the algorithm will not be presented due to page limit.

Verification of the reduced system

A dual shafted rotor is modelled with finite element method considering nonlinear effects from ball bearing and SFDs and mass unbalance excitation, Fig.1. The total degree of freedom of the system is 60. In numerical analysis, the variable-step Runge–Kutta–Fehlberg method is used. Figure 2 shows the relative error of the vertical displacement at the unbalance compared with the full-model solution using different *m* as the number of master modes (NMs), sided by the plot of computation time. Similar comparisons are also made for vibration energies. For the prescribed 10^{-3} of maximum relative error, the dashed area gives the hint on selection of *m* to best satisfy both computational efficiency and accuracy: the degree number of the reduced model can be chosen as much as one-third of the total degree number.

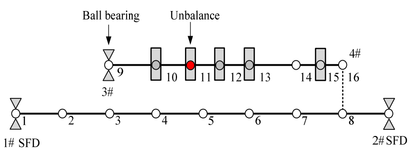


Fig.1 Computational model of the original system

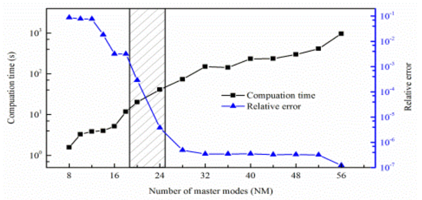


Fig. 2 Relative error (left ordinate) and computation time (right ordinate)

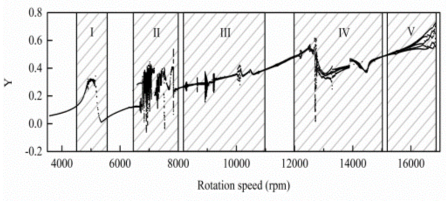
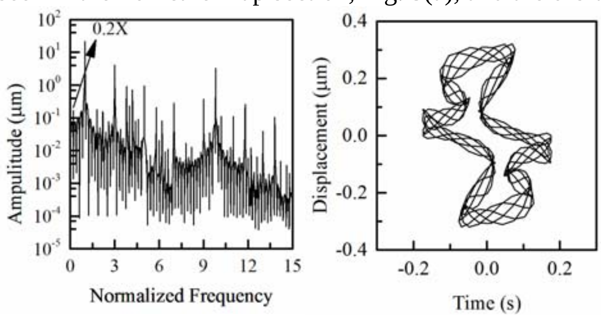


Fig.3 Bifurcation of the vertical displacement at Node 8.

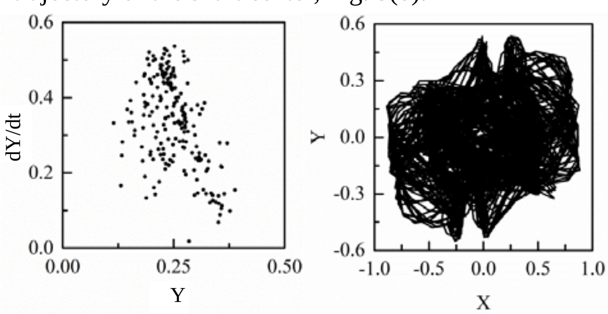
Results of the Nonlinear Dynamical Response

The response of the reduced system of the rotor is obtained numerically. The nonlinear behavior is examined in both time and frequency domains. Figure 3 shows the bifurcation in the vertical displacement at Node 8 with respect to the rotation speed. Multiple period doubling bifurcations can be found in the dashed areas I to V. Instability of the motion can be traced as the outcome bifurcation. The auto-power spectral density of displacement in Fig. 4(a) clearly reveals the multiple frequency components in the response that are fractional to the rotation speed. The orbital trajectory of the shaft center is shown in Fig. 4(b). As the rotation speed becomes higher, the motion of the rotor goes through several period doubling bifurcations and intermittency, and eventually becomes chaotic. The chaos in motion can be seen in the Poincare-map section, Fig. 5(a), and the orbital trajectory of the shaft center, Fig. 5(b).



(a) Power spectrum density (b) Orbital trajectory

Fig. 4 Motion response with rotation speed 5898rpm.



(a) Poincare-map section (b) Orbital trajectory

Fig. 5 Motion response with rotation speed 7835rpm.

Conclusions

In this paper, the effectiveness and efficiency of model reduction technique is demonstrated for a rotor-damper system through the nonlinear Galerkin method associated with approximate inertial manifold. Various nonlinear dynamical behaviours of bifurcation and chaos in the rotor motion are illustrated with the reduced model.

References

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