Nonlinear Dynamics of a Functionally Graded Nonlocal Nanobeam in Thermal Environment by using Incremental Harmonic Balance and Melnikov Method

Danilo Karličić and Milan Cajić
[*] *Mathematical Institute of the SANU,*
Kneza Mihaila 36, 11001 Belgrade, Serbia

Summary. In this paper, we investigate the nonlinear dynamics of a functionally graded (FG) nanobeam with geometric nonlinearity embedded in the Kelvin-Voigt viscoelastic medium. By using the D' Alembert principle, a nonlinear partial differential equation is obtained for transverse motion of FG nanobeam subjected to external and parametric excitations and thermal load. Bifurcations and rout to chaos are investigated by using the Galerkin and incremental harmonic balance method. Criteria of existence of chaos under the influence of different types of external excitation is given based on the Melnikov method. Moreover, effects of system parameters on the periodic and chaotic motions are investigated through several numerical examples.

Introduction

The functionally graded materials (FGM) are composed of at last two-phase inhomogeneous particulate composites, which are synthesized in such manner that volume fractions of constituents vary continuously along any desired spatial direction. This results in smooth variation of mechanical properties along a desired direction. Nazemnezhad et al. [1] have analyzed the free nonlinear vibration of FG nanobeam based on the von Karman deformation, Euler-Bernoulli beam theory and nonlocal elasticity. Ansari et al. [2] proposed nonlinear dynamic model to analyse the nonlinear forced vibration of FG nanobeam in thermal environment based on the surface elasticity theory. Yuda and Zhiqiang [3] analyzed bifurcation and chaos behavior of a thin circular FG plate in thermal environment by using the Melnikov method for two types of external excitations. The authors prove the existence of chaos by plotting the phase portraits and Poincare maps.

The mathematical models of nanostructures in thermal environment subjected to external and parametric excitation plays a crucial rule in the analysis and design of a new micro and nanoelectromechanical system. In the case when a FG nanobeam is under combined influences of time dependent axial and transversal loads, failure may occur at loads much smaller than those induced by static transversal or axial loads. The aim of this paper is to analyses the bifurcation and rout towards chaos of a FG nanobeam in thermal environment by using the IHB method and Melnikov method.

Using the D' Alembert's principle, nonlocal constitutive relation and Euler-Bernoulli beam theory, the governing equation of the embedded FG nanobeam in thermal environment (Fig. 1) can be expressed as:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\left[I_1\frac{\partial^2 w}{\partial t^2}-f(x,t)+kw+c\frac{\partial w}{\partial t}+(F_0+F_1\cos(\omega t))\frac{\partial^2 w}{\partial x^2}+N_T\frac{\partial^2 w}{\partial x^2}\right.$$
$$\left.-\frac{A_{11}}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2dx\frac{\partial^2 w}{\partial x^2}+\frac{B_{11}}{L}\left(\frac{\partial w(L,t)}{\partial x}-\frac{\partial w(0,t)}{\partial x}\right)\frac{\partial^2 w}{\partial x^2}\right]+\left(D_{11}-\frac{B_{11}^2}{A_{11}}\right)\frac{\partial^4 w}{\partial x^4}=0, \tag{1}$$

where $I_1=\int_A\rho(z)dA$ is the area mass density, $\{A_{11},B_{11},D_{11}\}=\int_A\frac{E(z)}{1-\nu(z)^2}\{1,z,z^2\}dA$ are the FG material parameters. By applying the Galerkin discretization and IHB method on the Eq. (1), we obtain the following incremental equation

$$[\boldsymbol{K}]\{\Delta\boldsymbol{a}\}=\{\boldsymbol{R}\}, \tag{2}$$

in which $[\boldsymbol{K}]$ is the coefficient matrix, $\{\boldsymbol{R}\}$ is the corrective vector, $\{\Delta\boldsymbol{a}\}$ is the vector of Fourier coefficients Δa_i and Δb_i. The set of linearized algebraic equations presented in Eq. (4) can be solved incrementally by using Newton – Raphson procedures as described in [4].

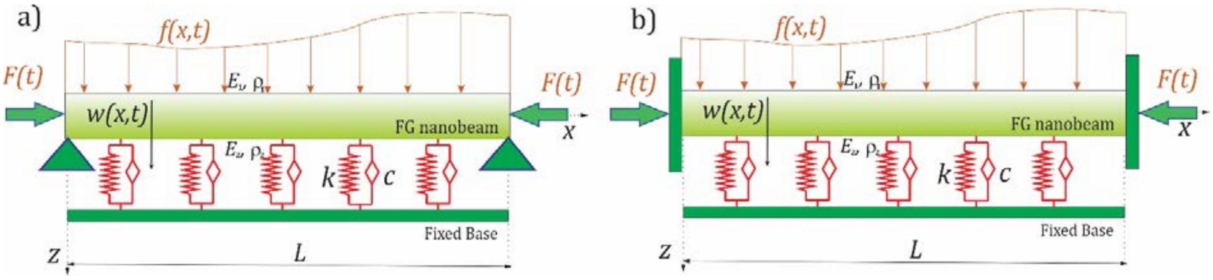

Fig. 1 The FG nanobeam embedded in a viscoelastic medium, a) simply-supported and b) clamped-clamped nanobeam.

Considering Melnikov theory for analyzing the homoclinic orbits and intersection of stable and unstable manifold [5], we obtain Melnikov function in the following form $\left(\delta=-\frac{\tilde{\kappa}}{2\omega^2},\beta=\frac{\omega_b^2}{\omega^2}\right)$,

$$M(t_0) = -\frac{\tilde{\mu}|\beta|^{\frac{3}{2}}}{3\omega\delta} + \frac{\sqrt{2}\tilde{\xi}\pi}{\omega^2}\sqrt{\frac{1}{\delta}}\,sech\left(\frac{\pi}{2\sqrt{|\beta|}}\right)sin(t_0) + \frac{8\pi\tilde{\lambda}}{\omega^2\delta}\,csch\left(\frac{\pi}{\sqrt{|\beta|}}\right)sin(2t_0). \quad (3)$$

Numerical results

Here, we analyze the periodic solution obtained by IHB method for combined excitation of the FG nanobeam for two types of boundary conditions, S-S and C-C, as shown in Fig. 2. After setting the values of dimensionless parameters, we can start with the incremental procedure as follows: I) we introduce the initial conditions for a_0 where other values of Fourier's coefficients are equal to zero; II) we apply the Newton – Raphson method for determination of incremental values of Fourier's coefficients $\boldsymbol{\Delta A}$ based on the Eq. (27), until residue Euclidian norm $|\boldsymbol{R}|$ is smaller than adopted tolerance 10^{-5}. Presented periodic solutions for buckled FG nanobeam is validated by using Runge - Kutta method, and we show excellent agreement.

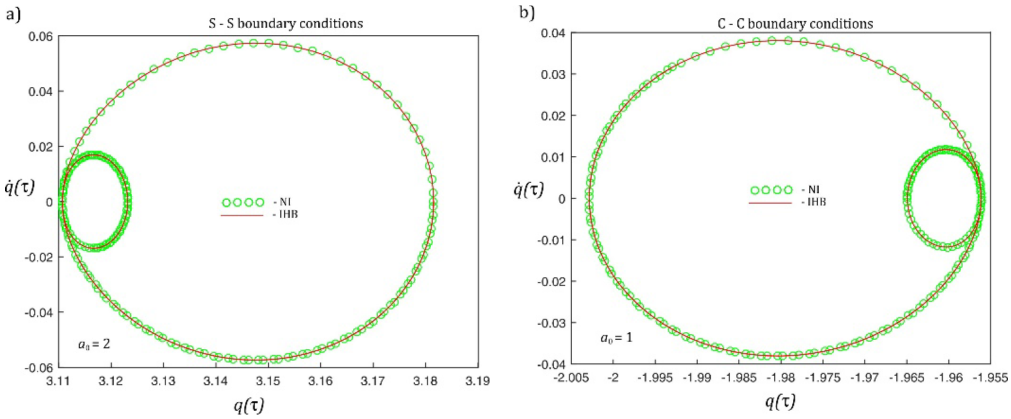

Fig. 2 Periodic behavior of FG nanobeam obtained from IHB method, a) S-S and b) C-C boundary conditions.

Based on the Melnikov function Eq. (3), the bifurcation curves for homoclinic orbits in $(\tilde{\xi}, \tilde{\mu})$ plane, where $\Delta T = 600\ K$, are show on the Fig. 3, in which $\tilde{\xi}$ is the amplitude of external load and $\tilde{\mu}$ is the damping ration. The region under the curves in the Fig. 3 represents chaotic regions.

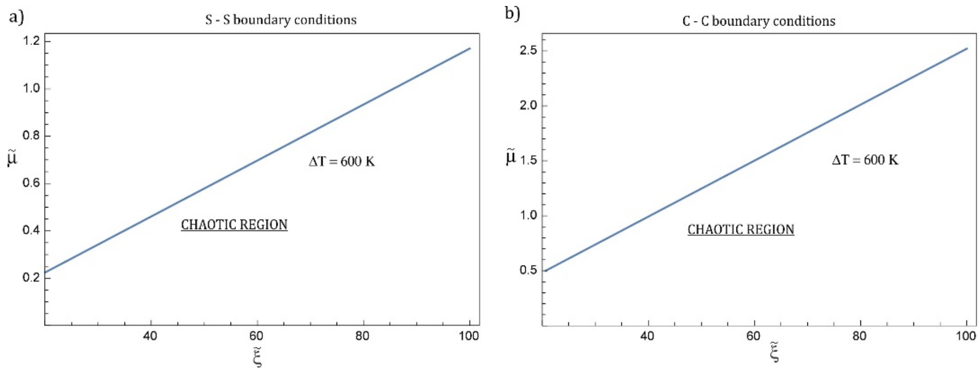

Fig. 3 Bifurcation curves for FG nanobeam obtained from Melnikov function, a) S-S and b) C-C boundary conditions.

Conclusions

It is shown that the IHB and the Melnikov method are very efficient technics to predict the periodic and chaos behavior in micro/nano-scale systems and devices.

***Acknowledgments*:** This research was supported by the research grants of the Serbian Ministry of Education, Science and Technological Development under the numbers OI 174001.

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