Modal Analysis of Structures in Periodic States

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Summary. A spectral method to compute Floquet forms of dynamical systems with periodically time-varying parameters is presented. Our algorithm is based on sorting of Hill matrix eigenvectors in the frequency domain. To make our point, we study the fundamental example of the oscillations of a 2D bi-articulated bar submitted to a periodic compressive load at its end. In addition to stability analysis, we compute modal information of the periodic system. The method of eigenvector sorting is compared to other spectral algorithms through stability analysis of a bi-articulated bars.

Context

Modal analysis is a valuable tool for engineers thanks to its computational efficiency and its ability to reveal intricate vibrations of structures in equilibrium state as well as their local stability. Surprisingly, to the best knowledge of the authors, the generalization of modal analysis to time-periodic systems has never been completely and clearly implemented. The goal of this research is to advance modal analysis by expanding its domain of applicability to systems in periodic state using Floquet theory. This means that time-periodic systems are decomposed into independent solutions, also known as Floquet forms (FFs) which are a time-periodic system's equivalents of modes. The difficulty of finding FFs is computing them in a time-invariant form [1]. We will show that it is possible to obtain FFs using a Floquet-Fourier-Hill transform [2], with the addition of an eigenvector sorting algorithm.

Method

In current state-of-the-art, time-periodic systems are analyzed either through temporal or spectral methods. Temporal methods typically integrate the monodromy matrix over time and are effective at finding stability, although they usually neglect modal information [3]. Spectral methods use Hill matrix with eigenvalue sorting algorithm [4] or without any sorting at all [5]. Again, modal information is overlooked.

FFs are obtained by sorting Hill matrix eigenvectors by looking at their vectorial weighted mean [6]. The eigenvector itself gives the time-periodic response envelope, the corresponding eigenvalue gives the stability and fundamental frequency of the FF. Combining the eigenvalues and vectors gives the FF which is used in transient analysis. This time-periodic modal analysis is particularly powerful since after calculating the FFs no time integration is needed to find the system's state.

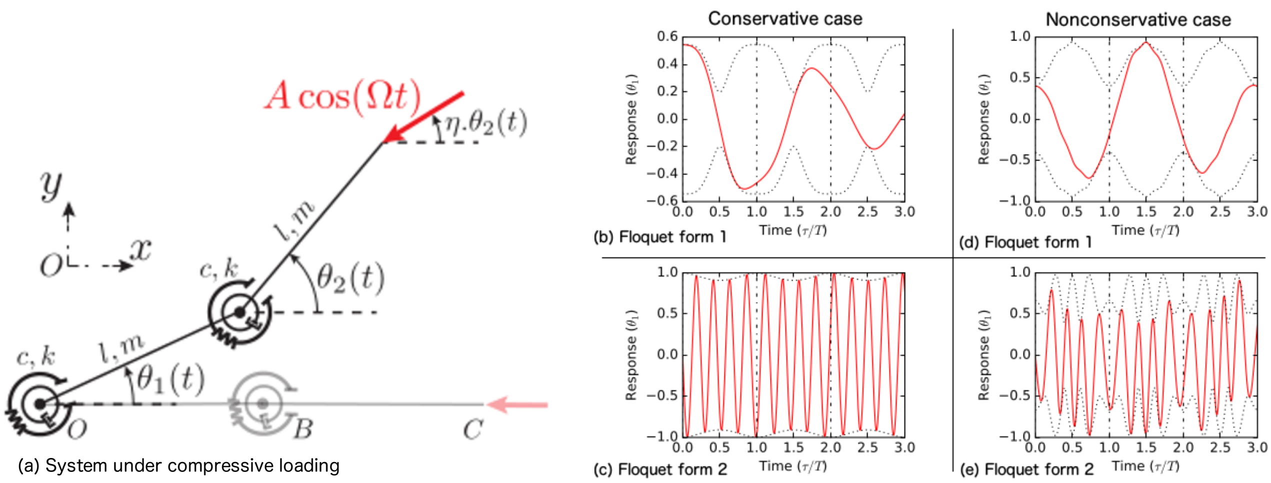

Figure 1: (a) Bi-articulated bar under compressive loading, either conservative ($\eta = 0$) or non-conservative ($\eta = 1$). The red lines show actual response, the eigenfunction or envelope is shown in grey dotted lines. The verical dashed lines represent each period. (b-e) Floquet form response of θ_1 over dimensionless time. With load $\lambda = 0.5\lambda_{crit}$, at frequency $\Omega = 1.55\omega_1$

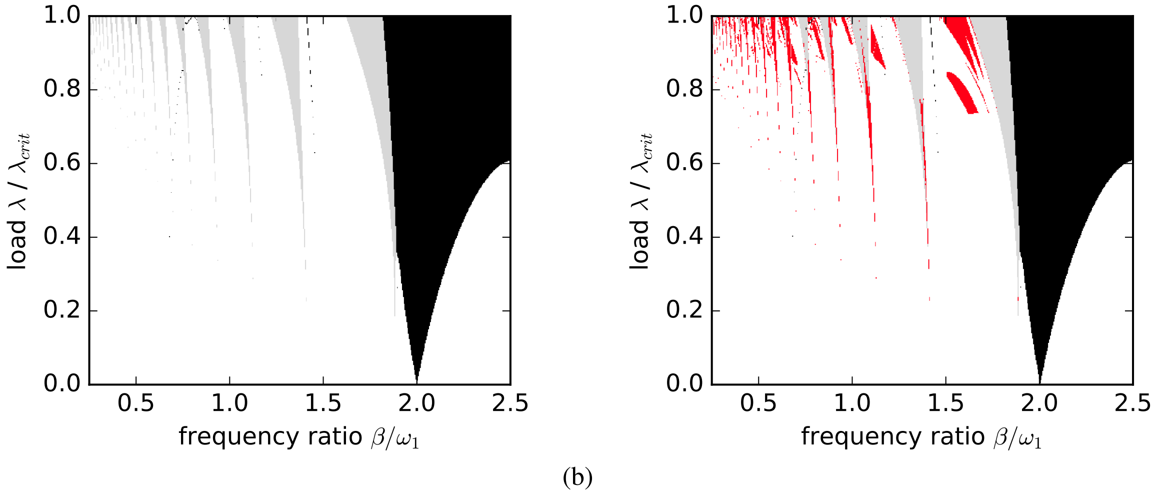

Figure 2: Stability charts in parameter space (a,b) of the non-conservative system. Grey regions show Neimark-Sacker instability, black regions illustrate period-doubling instabilities. Techniques used are (a) eigenvector sorting [6] (b) eigenvalue sorting [4]. On the x-axis β/ω_1 represents the dimensionless excitation frequency over the first dimensionless natural frequency. On the y-axis the load ratio is represented. Red regions show where eigenvalue sorting is not yet converged.

Application

To illustrate modal analysis of periodic systems, an example of a bi-articulated bar under periodic compressive loading is investigated, see Fig.1(a). The load direction is either conservative ($\eta = 0$) or non-conservative ($\eta = 1$), this leads to rich instability behaviour. Using this fundamental example FFs are computed and analyzed for stability and transient response. This is compared to eigenvalue sorting for stability where it is shown that in the non-conservative case eigen vector sorting requires less harmonics to decompose the problem. A big advantage of the current method is the availability of modal information in time-invariant form, making transient response computationally efficient once FFs are found.

Fig.1 shows the Floquet forms of the bi-articulated bar in the conservative and non-conservative case. The envelope is periodic with respect to time and the response is quasi-periodic. The envelopes, or eigenfunctions, of the FFs are modulated in both cases, in the conservative case the first FF is more modulated than the second FF as shown in Fig.1(b,c). In case of nonconservative loading the response and envelope of both FFs are strongly modulated as can be seen in Fig.1(d,e)

Using the eigenvalues found from Hill's matrix, stability of the system is analyzed in parameter space as shown in Fig.2. The Floquet exponent gives the growth/damping rate and the frequency of the system. Instabilities can be classified based on the frequency of instability. The non-conservative case shows period-doubling as well as Neimark-Sacker bifurcation. In the conservative case (not shown here) we can see symmetry-breaking bifurcation as well as period-doubling bifurcations. The red areas show where eigenvalue sorting gives erroneous (not yet converged) results, while the eigenvectors and eigenvalues come from the same Hill matrix. This shows that the vector sorting method is computationally more efficient than eigenvalue sorting.

Future work

The current case of a 2-DoF problem paves the way to extending time-periodic modal analysis of an arbitrary number of degrees of freedom using Finite Element Analysis. This would make analysis of time-periodic systems much more accesible to engineers in their daily work, improving designs of structures in periodic state.

References

[1] Sinha S.C., et al. (1998) Construction of Dynamically Equivalent Time-Invariant Forms for Time-Periodic Systems *Nonlinear Dynamics* **16**:203-221

[2] Deconinck B., Kutz J.N. (2006) Computing spectra of linear operators using the Floquet-Fourier-Hill method *J. Comp. Physics* **219**:296-321

[3] Peletan L., et al. (2013) A comparison of stability computational methods for periodic solution of nonlinear problems with application to rotordynamics. *Nonlin. Dyn.* **72**:671-682

[4] Zhou J., et al. (2004) Spectral characteristics and eigenvalues computation of the harmonic state operators in continuous-time periodic systems *Systems and Control Letters* **53**:141-155

[5] Lee C. W. et al. (2007) Modal analysis of periodically time-varying linear rotor systems. *J. Sound and Vibration* **303**:553-574

[6] Lazarus A., Thomas O. (2010) A harmonic-based method for computing the stability of periodic solutions of dynamical systems. *CR Méca.* **338**:510-517