On perturbations methods and their applicability in the study of vibrations of axially moving strings and beams

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Summary. In this paper the transversal vibrations of axially moving strings or beams with constant or time-varying lengths, time-varying velocities, and/or time-varying tensions are studied. By applying perturbation methods, asymptotic approximations of the solutions of the formulated initial-boundary value problems are constructed. For conveyor belt problems it will be shown how the multiple timescales perturbation method in combination with the method of characteristic coordinates can be used. Also the applicability of the truncation method for these types of problems is discussed. Further, resonances and vibrations in an elevator cable system due to wind-induced building sway, will be described by using regular and singular perturbation techniques.

Introduction

Many engineering devices contain axially moving continua, and in many applications such as conveyor belts, elevator cables, paper sheets, satellite tethers, crane and mine hoist, and cable-driven robots, we encounter in parts of the systems constant or variable lengths, or constant or variable transport speeds during operation. The reader is referred to [1-14] for all kinds of examples. To improve the design of elevators, one of the major tasks is to develop a better understanding of elevator cable dynamics and to develop new methods to effectively reduce the vibration and noise. The dynamics of media with variable-length, -velocity and -tension is the subject of this paper. Due to small allowable vibrations the lateral and vertical cable vibrations in elevators can be assumed to be uncoupled and only lateral vibrations are considered here. The elevator car is modelled as a rigid body of mass m attached at the lower end of the cable, and the suspension of the car against the guide rails is assumed to be rigid, where the external excitation due to wind is also considered at the boundary. We consider a basic and a simple, asymptotic string or beam model for an elevator cable. The initial boundary value problems will be studied, and explicit asymptotic approximations of the solutions, which are valid on a long timescale, will be constructed. Two cases for varying-length are considered, (i) $l(t) = l_0 + vt$ with boundary excitations $u(0,t) = \alpha \sin(\Omega t)$, where l_0 is the initial cable length, v denotes the constant cable velocity, α is the oscillation amplitude of the building from equilibrium and Ω is the frequency of this excitation and (ii) $l(t) = l_0 + \beta \sin(\omega t)$, where β defines a length variation parameter and ω is the angular frequency of the length variation, and $l_0 > |\beta|$. For the harmonic length-variations, it will be shown that the truncation method cannot be applied in order to obtain asymptotic results on long timescales, and for boundary excitations and linear length variations, the interesting phenomena of autoresonance, see for instance [5,7,13] will be discussed in detail. This phenomena arise when there occurs a passage through a dynamic resonance. It will also be shown that an $O(\varepsilon)$-amplitude excitation gives rise to $O\left(\sqrt{\varepsilon}\right)$ responses. For further details we refer the reader to [6,7,13,14]. For conveyor belt problems it will be shown how the two timescales perturbation method in combination with the method of characteristic coordinates can be used to construct asymptotic approximations of the solutions on long timescales. Also for these conveyor belt problems it turned out that the truncation method was not applicable in the string-like case to obtain asymptotic results on long timescales. For details of the computations we refer the reader to [8,9].

Mathematical Models

Relative to the fixed coordinate system the lateral displacement of the string or beam particle instantaneously located at spatial position x at time t, where $0 \le x \le l(t)$, is described by $u(x,t)$. The equations of motion for a vertically moving string with time-varying length, velocity and tension are formulated and are given by

$$\rho\left(u_{tt} + 2vu_{xt} + \dot{v}u_x + v^2 u_{xx}\right) - \left(P(x,t)u_x\right)_x = 0, \quad t > 0, \quad 0 < x < l(t),$$
$$u(0,t) = 0 \quad \left(\text{or } \alpha \sin(\Omega t)\right), \quad u\left(l(t),t\right) = 0, \quad t > 0,$$
$$u(x,0) = f(x), \quad \text{and} \quad u_t(x,0) = h(x), \quad 0 < x < l(0),$$

where the subscript for u denotes partial differentiation,
$P(x,t) = mg + \rho\left(l(t) - x\right)g - m\dot{v} - \rho\left(l(t) - x\right)\dot{v}$ is an axial force arising from its own weight and longitudinal acceleration, α is the excitation amplitude at the upper end, Ω is the excitation frequency, g is the acceleration due to gravity, and where $f(x)$ and $h(x)$ represent the initial displacement and the initial velocity, respectively.

Conclusions

A multiple timescales perturbation method has been used in search of infinite mode approximate solutions. It is observed that there are applicability issues with the truncation method for string-like problems. A set of new problems have arisen in studying the elevator cable system under weak boundary excitations which is related to the investigation of a physical phenomenon known as autoresonance. All detailed computations for the aforementioned problems can be found in [1, 6-14].

References

[1] Sandilo, S.H. and van Horssen, W.T. (2012) On boundary damping for an axially moving tensioned beam. *ASME, J. Vib. Acoust*, 011005: 011005.

[2] Zhu, W.D. and Xu, G.Y. (2003) Vibration of elevator cables with small bending stiffnes. *J.Sound Vib.*, 699: 699-679.

[3] Zhu, W.D. and Chen Y. (2005) Forced response of translating media with variable length and tension: application to high-speed elevators. *Proc. Inst. Mech. Eng. K.J. Multi-body Dyn.*, 53: 53-35.

[4] Kaczmarczyk S. (1997) The passage through resonance in a catenary-vertical cable hoisting system with slowly varying length. *J. Sound Vib.*, 265: 265-243.

[5] Frajans, J. and Friedland, L. (2001) Autoresonant (non-stationary) excitation of pendulums, plutinos, plasmas, and other nonlinear oscillators. *Am. J. Phys.*, 1102: 1102-1096.

[6] Sandilo, S.H. and van Horssen, W.T. (2014) On variable length induced vibrations of a vertical string. *J. Sound Vib.*, 333: 2432-2449.

[7] Sandilo, S.H. and van Horssen, W.T. (2015) On a cascade of autoresonances in an elevator cable system. *Nonlinear Dynamics* 80(3): 1613-1630.

[8] Malookani, R.A. and van Horssen, W.T. (2016) On the asymptotic approximation of the solution of an equation for a non-constant axially moving string. *J. Sound Vib.*, 367: 203-218.

[9] Malokani, R.A. and van Horssen, W.T. (2015) On resonances and the applicability of Galerkin's truncation method for an axailly moving string with time-varying velocity. *J. Sound Vib.*, 344: 1-17.

[10] Malokani, R.A. and van Horssen, W.T. (2017) On parametric stability of a non-constant axially moving string near resonances. *J. Vibr. Acoust.* 139(1); DOI: 10.1115/1.4034628.

[11] Gaiko, N.V. and van Horssen, W.T. (2016) On wave reflections and energetics for a semi-infinite traveling string with a nonclassical boundary support. *J. Sound Vib.*, 370: 336-350.

[12] Gaiko, N.V. and van Horssen, W.T. (2015) On the transverse, low frequency vibrations of a travelling string with boundary damping. *J. Vibr. Acoust.*137(4); DOI: 10.1115/1.4029690.

[13] Gaiko, N.V. and van Horssen, W.T. (2016) Resonances and vibrations in an elevator cable system due to boundary sway (submitted for publication).

[14] Gaiko, N.V. and van Horssen, W.T. (2016) On transversal oscillations of a vertically translating string with small time-harmonic length variations. *J. Sound Vib.*, 383: 339-348.