Forced Resonance Vibrations of the Dissipative Spring-Pendulum System

Yuri Mikhlin[*] and Kateryna Plaksiy[*]

[*]*Department of Applied Mathematics, National Technical University "KhPI", Kharkov, Ukraine*

Summary. Dynamics of the dissipative spring-pendulum system under periodic external excitation in the vicinity of external resonance and simultaneous external and internal resonances is studied. The concept of nonlinear normal vibration modes is used in this analysis. The multiple scales method and subsequent transformation to the reduced system with respect to the system energy, an arctangent of the amplitudes ratio and a difference of phases of required solutions are applied. Transient nonlinear normal modes, which exist only for some particular levels of the system energy, are obtained. In the vicinity of values of time, corresponding to these energy levels, the transient modes temporarily attract other system motions. Interaction of nonlinear vibration modes under resonance conditions is also analysed.

Introduction

Resonances cause complex behavior of the nonlinear system. Different resonance effects are analyzed in numerous publications, in particular, in books [1-4]. The investigation of energy transfer and localization are presented for different nonlinear systems in these and other publications. Nonlinear normal modes (NNMs) are important element of the nonlinear systems behavior. The Kauderer-Rosenberg concept of NNMs, first proposed for conservative systems [5], is based on the determination of trajectories in the nonlinear system configuration space. Theory of NNMs for conservative and non-conservative systems and different applications of this theory are presented in numerous publications, in particular, in [6-8]. In nonlinear dissipative systems the classical NNMs by Kauderer-Rosenberg cannot appear due to exponential decrease of vibration amplitudes, but some similar vibration regimes exist. The so-called *reduced system*, which was first used for nonlinear conservative systems [9], permits to show some important elements of nonlinear dissipative system resonance behavior. The reduced system is written with respect to the system energy, the arctangent of the ratio of amplitudes and the difference of phases. The interaction of vibration modes, their stability, localization of energy and other characteristics of the resonance behavior can be analyzed by such systems. The reduced system was used in analysis of resonance behavior of some nonlinear dissipative systems in papers [10,11]. Besides, the *transient nonlinear normal modes* (TNNMs) existing only for some specific values of the system energy, were first described in these publications. Although TNNMs disappear when the energy level decreases, they temporarily attract other motions of the dissipative system, when the system energy is close to these specific energy values. Here the spring-pendulum system (the rotator on the spring) under external periodic excitation is investigated for the cases of two external resonances on the fundamental frequencies and for the case of simultaneous external and internal resonances.

Different forced and internal resonance in vibrations of dissipative spring-pendulum system

The spring-pendulum system (the rotator on the spring) with small dissipation, under external periodic excitation (Fig. 1), is considered. The external resonance on the fundamental frequency corresponding to the spring vibrations is analyzed. The multiple time scales method is used; additional change of variables gives the *reduced system,* written with respect to the energy characteristic parameter *K*, the arctangent of the ratio of amplitudes ψ and the difference of phases φ. Trajectories of the reduced system in the space (ψ,φ) are presented in Fig. 2 for some system parameters. All trajectories approach the line $\psi = 0$ which corresponds to the vibration mode with localization on the spring. This result is confirmed by construction of trajectories in the system configuration space (Fig.3), where $u = y/R$.

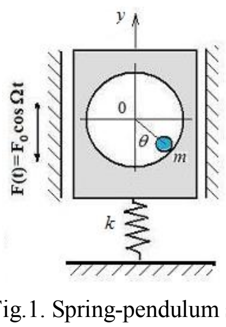

Fig.1. Spring-pendulum system

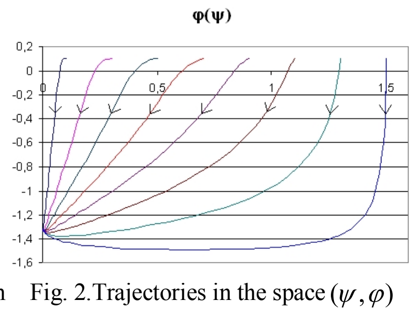

Fig. 2.Trajectories in the space (ψ,φ)

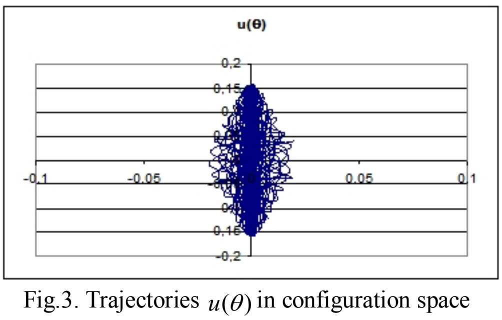

Fig.3. Trajectories $u(\theta)$ in configuration space

In the case of simultaneous external and internal resonances corresponding analysis by using the reduced system gives the resonance behavior which is shown in Fig.4.

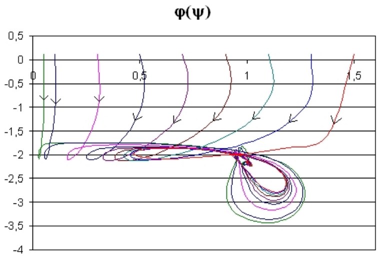

Fig.4. Trajectories in the space (ψ,φ)

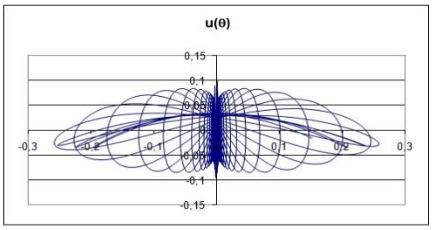

Fig. 5a. Trajectories $u(\theta)$ for $\tau \in [0,250]$

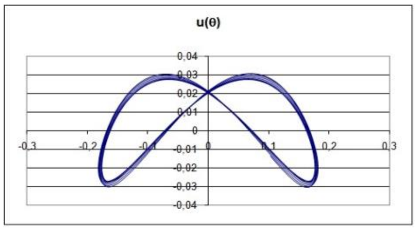

Fig. 5b. Trajectories $u(\theta)$ for $\tau \in [400,500]$

Each trajectory has a loop near some quasi-equilibrium state of the reduced system. This state moves in the space (ψ,φ) and corresponds to the *transient nonlinear normal mode* (TNNM) [10,11], which exists only for specific value of the system energy. So, TNNM exists in some moment of time corresponding to this energy level. It is important, that these transient modes are temporarily attractive and other motions of a system are close to some TNNM near the mentioned moment of time. We can see in the Fig. 4, that later, when the TNNM disappears, trajectories in the plane (ψ,φ) approach the equilibrium position which corresponds to the stable nonlinear normal mode of coupled vibrations. This equilibrium position is closer to the straight line $\psi = \pi/2$, which corresponds to localized on pendulum vibrations, than to the straight line $\psi = 0$, which corresponds to localized vibrations of spring. So, the localized vibrations of spring are not stable.

Analysis of obtained results permits to conclude that under simultaneous external and internal resonances the TNNM of coupled vibrations appears. At the beginning of the process localized mode of spring vibrations loses stability and motions of the system become close to this TNNM which is determined by trajectories, which are close to parabola with branches down (Fig.5a). Then, due to instability of this mode, motions tend to the mode of coupled vibrations which is stable in the resonance region (Fig.5b). This stable mode is close to the localized mode of the pendulum vibrations, and it can be used in the problem of vibration absorption. Namely, it is possible to guarantee transfer from vibrations of spring to the pendulum vibrations, where the vibration energy can be dissipated.

In the case of external resonance on the frequency of pendulum vibrations two variants are possible. For large amplitudes of external excitation trajectories in the space (ψ,φ) approach to the straight line $\psi = \pi/2$, which corresponds to the mode with large amplitudes of pendulum vibrations; for small amplitudes of the external excitation trajectories approach to the straight line $\psi = 0$, which corresponds to localization of the energy on the spring. In this region of the system parameters localized vibrations of the spring stay stable and large amplitudes pendulum vibrations are not observed. This result is confirmed by construction of trajectories in the system configuration space.

Conclusions

Dynamics of the dissipative spring-pendulum system under the periodic external excitation is analyzed in the vicinity of two external resonances on the fundamental frequencies and in the vicinity of simultaneous external and internal resonances. Analysis of the resonance dynamics is made by using the concept of nonlinear normal modes and by transform to the *reduced system*. The localized vibration modes and the modes of coupled vibrations are obtained. Transfer from unstable vibration modes to the stable ones is described. In the vicinity of the resonance the *transient nonlinear normal modes* (TNNMs), which exist only for some levels of the system energy, appear. The TNNM temporarily attracts other motions. The obtained results can be useful in problem of the elastic vibrations extinguishing with the help of nonlinear absorbers.

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