Resonance phenomena in a two-layer shear flow interacting with two vortices in bottom layer

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Summary. The paper deals with a dynamical system governing the motion of two point vortices embedded in the bottom layer of a two-layer rotating flow experiencing linear deformation and their influence on fluid particle advection in the upper-layer. If the deformation is stationary, the vortices can move periodically in a bounded region. This vortex periodic motion plays the role of a perturbation to the fluid particle dynamics. Due to periodic nature of the perturbation, a vast spectrum of resonance phenomena appears. Analyzing the conceivable resonances observable in the upper-layer of the flow is the main goal of the study.

We study the following systems of nonlinear ordinary differential equations governing the positions of the vortices and a passive scalar. The governing equations for a scalar in the upper-layer ensue [1]:

$$\begin{aligned}\frac{dx}{dt} &= 2y(S-\Omega) - \frac{H_2}{H}\sum_{i=1}^{2}\frac{(y-y_i)\mu_i}{R_i}\left[\frac{1}{R_i} - K_1(R_i)\right],\\ \frac{dy}{dt} &= 2x(S+\Omega) + \frac{H_2}{H}\sum_{i=1}^{2}\frac{(x-x_i)\mu_i}{R_i}\left[\frac{1}{R_i} - K_1(R_i)\right],\end{aligned} \tag{1}$$

where S is the shear coefficient, Ω is the rotation coefficient, H_1, H_2 are the layers' depths, $H = H_1 + H_2$ is the total depth, K_1 is the modified Bessel function of the first order and $R_i = \left((x-x_i)^2 + (y-y_i)^2\right)^{1/2}$.

By substituting the corresponding vortex coordinates in eq. (1) and excluding self-induction, one readily obtains the governing equations for the vortex trajectories

$$\begin{aligned}\frac{dx_\alpha}{dt} &= 2y_\alpha(S-\Omega) - \frac{1}{H}\frac{\mu_{3-\alpha}}{r_{12}}(y_\alpha - y_{3-\alpha})\left[\frac{H_2}{r_{12}} + H_1 K_1(r_{12})\right],\\ \frac{dy_\alpha}{dt} &= 2x_\alpha(S+\Omega) + \frac{1}{H}\frac{\mu_{3-\alpha}}{r_{12}}(x_\alpha - x_{3-\alpha})\left[\frac{H_2}{r_{12}} + H_1 K_1(r_{12})\right],\end{aligned} \tag{2}$$

where $r_{12} = \left((x_1-x_2)^2 + (y_1-y_2)^2\right)^{1/2}$.

Figure 1a shows a typical phase portrait of the vortex motion. Figure 1b shows how the frequency of the vortex motion changes depending on the vortex initial positions. This frequency is the characteristic frequency of the perturbation influencing the dynamics of the passive scalars in the both layers.

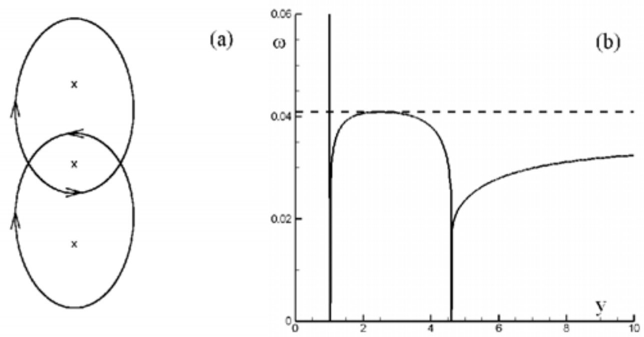

Figure 1: (a) Typical vortex trajectories. (b) Typical frequency of the vortex motion.

When the vortices are stationary, they induce steady-state passive scalar advection. Figure 2 demonstrates the separatrices of all the possible steady-state configurations. When we shift vortices from elliptic points, the upper-layer dynamics changes drastically. Figure 3 demonstrates a Poincare section for the upper-layer fluid particles for the case of rotating vortices.

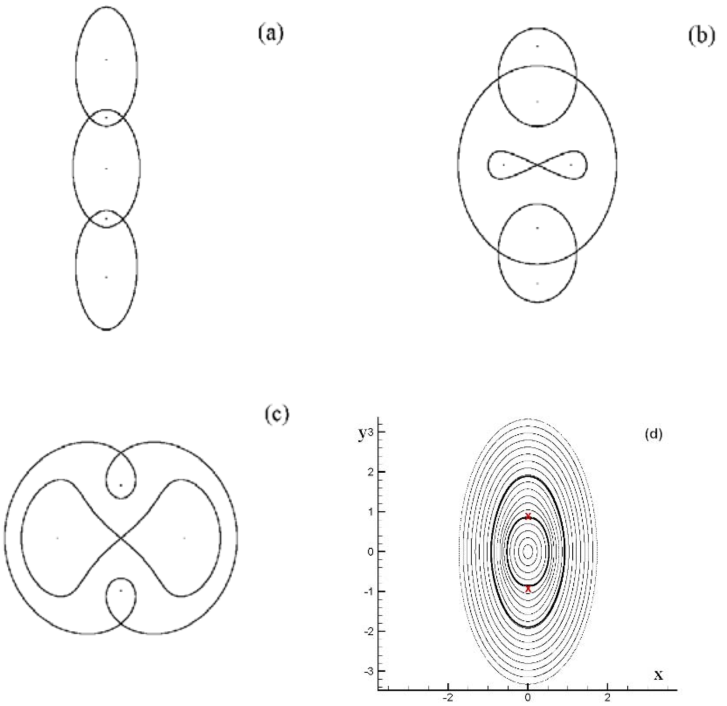

Figure 2: Separatrices of the steady-state passive scalar advection in the upper-layer depending on the shear parameter. $\mu_1 = \mu_2 = 0.3$, $\Omega = -0.02$: (a) $S = -0.015$, (b) $S = -0.007$, (c) $S = -0.001$ and the phase portrait for week vortices $\mu_1 = \mu_2 = 0.05$, $S = -0.01$.

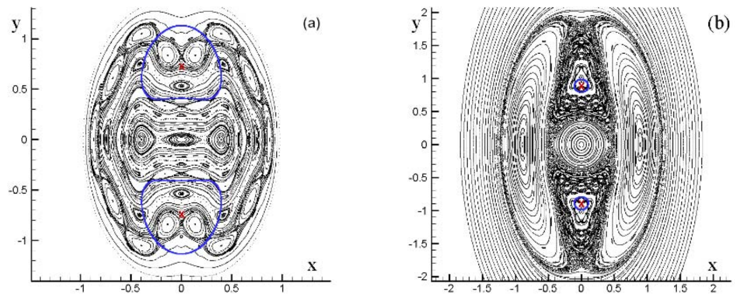

Figure 3: Poincare sections of fluid particle trajectories in the upper-layer governed by the perturbed system (1). The perturbation is implemented as a deviation δ from the stationary configuration in the starting vortex positions. The crosses indicate the stationary positions of the vortices in the bottom layer. The bold curves encircling the crosses are the trajectories of the vortices in the bottom layer. The system parameters are $S = -0.01$, $\Omega = -0.02$ and (a) $\mu_1 = \mu_2 = 0.03$, $\delta = 0.4$ and vortex rotation frequency is $\omega_\delta = 0.039829$; (b) $\mu_1 = \mu_2 = 0.0475$, $\delta = 0.1$, $\omega_\delta = 0.04329$.

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References

[1] E. A. Ryzhov. K.V. Koshel. Global chaotization of fluid particle trajectories in a sheared two-layer two-vortex flow. *CHAOS*. V. 25, n. 103108. 2015.