Nonlinear dynamical response of fluid conveyed thin-walled piezoelectric cylindrical shell

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Abstract

Summary. The vibration and instability of a thin-walled smart cylinder with internal fluid flow is analyzed. The mathematical modeling is carried out based on nonlinear Donnell's shell theory and scalar potential function. The governing equations are obtained using energy methods and modal expansion analyses which are then solved using Eigen value problem and numerical integration. In the numerical results section the effects of various parameters such as mean flow velocity, aspect ratio, temperature change and excitation frequency is studied in details. It is hoped that the results of this study play an important role to design new instability alert sensors for fluid conveying pipes.

Keywords

Smart Materials, Nonlinear Vibration, Fluid Flow, Cylindrical Shell, Harmonic Excitation

Introduction

In recent years, piezoelectric materials have attracted more attention due to their interesting applications to smart structures such as sensors and actuators. This is due to the coupling effects between electrical and mechanical fields of piezoelectric materials. In fact by applying mechanical field, induced voltage is achieved and vice versa.

On the other hands, dynamic behavior of the structures in contact with the fluid flows have been attracted more attentions in recent years due to their wide applications in many engineering fields. In this regard, Amabily et al. studied the dynamic behavior and stability of fluid conveyed shell [1]. Reddy and Wang [2] are analyzed the dynamics of beams containing fluid flow using finite element method. In this paper, dynamical stability and vibration analyses of a thin walled piezoelectric shell under internal fluid flow and external harmonic load is investigated using energy methods. The results of this paper especially may be used for measurement and stability control of fluid conveyed pipes (i.e. instability alert sensors).

Piezoelasticity theory

The constitutive equations for the piezoelectric materials are given by

$$\begin{Bmatrix} \sigma \\ D \end{Bmatrix} = \overbrace{\begin{bmatrix} C^E & -e \\ e^T & \in^{\varepsilon} \end{bmatrix}}^{\mathrm{C}} \begin{Bmatrix} \varepsilon \\ E \end{Bmatrix} - \begin{Bmatrix} \lambda \\ p \end{Bmatrix} \Delta\Theta, \tag{1}$$

where $\{\sigma\},\{\varepsilon\},\{D\}$ and $\{E\}$ are stress, strain, electric displacement and electric field vectors, respectively, and $[C],[e]$ and $\{\in\}$ are matrices of elastic stiffness, piezoelectric and dielectric constants, respectively. Furthermore, the coefficients of thermal expansion, pyroelectric and temperature change are shown by $\{\lambda\},\{p\}$ and $\Delta\Theta$, respectively. The component of the electric field in Eq. (1) is expressed in terms of electric potential function $E = -\nabla\varphi$.

Method of solution

In this study, using Lagrange equations of motion a set of nonlinear governing equations of motion are obtained as

$$[M]\{\ddot{q}\} + [D]\{\dot{q}\} + [K]\{q\} = \{F\}. \tag{3}$$

where $[M]$ and $[D]$ are the mass and damping matrices, respectively and $[K]$ is the stiffness matrix composed of linear and nonlinear terms as

$$[K] = [K_L + K_{NL}]. \tag{4}$$

The Eq. (3) can be re-written as

$$\begin{bmatrix} M_{dd} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{q}_d \\ \ddot{q}_\varphi \end{Bmatrix} + \begin{bmatrix} D_{dd} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_d \\ \dot{q}_\varphi \end{Bmatrix} + \begin{bmatrix} K_{dd} & K_{d\varphi} \\ K_{\varphi d} & K_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} q_d \\ q_\varphi \end{Bmatrix} = \begin{Bmatrix} F_d \\ F_\varphi \end{Bmatrix}. \tag{5}$$

Due to existence of static coupling between mechanical and electric displacement, the parts of mass and damping matrices associated with $\dot{q}_\phi$ and $\ddot{q}_\phi$ are obtained zero. By means of second set of equations of Eq. (6), the amplitude of electric field vector can be calculated in terms of displacement vector:

$$\{q_\phi\} = -\left[K_{\phi\phi}{}^{-1} K_{\phi d}\right]\{q_d\}. \tag{7}$$

Eliminating q_ϕ in Eq. (6) and using Eq. (7), yields the modified equations of motion which is then solved and analyzed using the state space method, forth order numerical integration and energy spectrum analyses.

Numerical results

The variations of dimensionless natural frequency versus dimensionless flow velocity for first four modes are shown in Fig. 1. It can be found that the imaginery part of frequency is decreased with increasing flow velocity for all modes until reaching the value zero and instability is occurred in the system. The flow velocity in this point is defined as critical flow velocity that is located at $U_f^* = 0.0044$ for the first vibration mode. Within the zero-frequency area of the first mode $(0.0044 \le U_f^* \le 0.0089)$, the real part of complex frequency is increased as shown in Fig. 2. Furthermore the magnitudes of natural frequencies of the first and second modes are merged within $0.0089 \le U_f^* \le 0.0185$ which is physically known as flutter instability phenomenon. The behavior of the system in the other modes is similar.

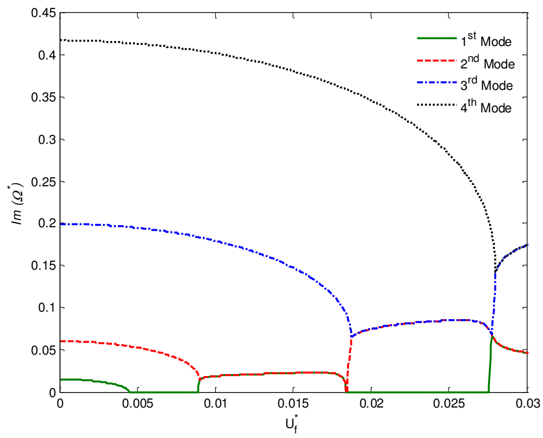

Fig. 1. Dimensionless natural frequencies versus dimensionless flow velocity for 1st to 3rd mode.

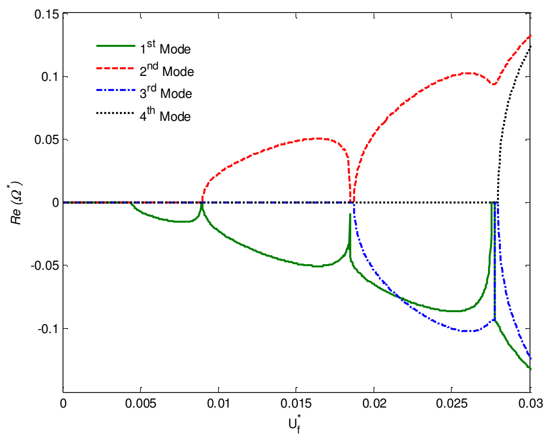

Fig. 2. Dimensionless damping frequencies versus dimensionless fluid

Conclusion

The most important results of this paper are listed below

1. Increasing the flow velocity, the natural frequencies for all modes decreased until reaching the divergence instability point (i.e. $\mathrm{Im}(\Omega^*) = 0$ and $\mathrm{Re}(\Omega^*) \neq 0$)
2. For the range $0.0089 \le U_f^* \le 0.0185$ the value of first and second modes are merged which is called flutter instability.
3. The induction electric potential is increased by increasing the flow velocity which is due to increasing strain field of the shell. Hence this fact may be considered to design new instability alert sensors.
4. Increasing the temperature increased smoothly the vibration amplitude of the system.

References

[1] Amabili, M., 2007. *Nonlinear Vibrations and Stability of Shells and Plates*. Cambridge University press, University of Parma, Italy.

[2] Reddy, J.N., Wang, C.M., 2004. *Dynamics of Fluid conveying Beams: Governing Equations and Finite Element Models*. Centre for Offshore Research and Engineering, National University of Singapore.