Active Vibration Control of a Nonlinear System using Pole Placement

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Summary. Active vibration control of a nonlinear system is presented using pole placement technique. The receptance method is applied to design a feedback control system to suppress vibrations of a multi degree-of-freedom (MDOF) nonlinear system. Both velocity and displacement feedback are implemented to change the damping and the frequency of the resonant peak. The feedback control gains are obtained from an iterative method as the input amplitude varies as a result of the control force. The closed-loop amplitude of the response demonstrates the successful assignment of the peak resonance. The forcing amplitude is then varied and the level dependent feedback control gains, which can assign the peak resonant at the prescribed location are obtained.

Application of the Receptance Method to a MDOF Nonlinear System

Active control is widely used to control the vibrations of linear systems [1]. However, there is not much work in the field of active vibration control of nonlinear systems. The problem is challenging, since the response of the nonlinear system is amplitude dependent, and any changes in the input amplitude can affect the performance of the closed-loop system. Therefore, to maintain the performance, the control system should take into account this level dependency on the input forcing. The receptance method, which is based on the frequency responses has been developed for a single DOF system to assign the peak resonance [2]. Here, the receptance method is extended to a MDOF nonlinear system to assign the peak resonances, while maintaining the other modes unchanged. The control force distribution is chosen to be orthogonal to the unchanged mode.

The dynamic equations for a MDOF nonlinear system can be written as,

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)+\mathbf{f}_N(\mathbf{x},\dot{\mathbf{x}})=\mathbf{f}_{ext}(t)+\mathbf{b}u(t) \tag{1}$$

where, $\mathbf{M},\mathbf{C},\mathbf{K}$ are the mass, damping and stiffness matrices, respectively. $\mathbf{x}(t)$ is the displacement responses, $\mathbf{f}_{ext}(t)$ is the external excitation, $\mathbf{f}_N(t)$ is the nonlinear forcing vector. The control force is a linear combination of the velocities and displacements, with a control force distribution $\mathbf{b}$ and displacement and velocity feedback gains $\mathbf{g}$ and $\mathbf{f}$ respectively, such that,

$$u(t)=-\mathbf{g}^T\mathbf{x}(t)-\mathbf{f}^T\dot{\mathbf{x}}(t) \tag{2}$$

The closed-loop response amplitudes can be written in terms of the open-loop response amplitudes and the feedback gains using the Sherman-Morrison formula in the frequency domain as discussed in [2].

$$\hat{\mathbf{H}}(s,\mathbf{X})=\mathbf{H}(s,\mathbf{X})-\frac{\mathbf{H}(s,\mathbf{X})\mathbf{b}(\mathbf{g}+s\mathbf{f})^T\mathbf{H}(s,\mathbf{X})}{1+(\mathbf{g}+s\mathbf{f})^{T^T}\mathbf{H}(s,\mathbf{X})\mathbf{b}} \tag{3}$$

The open-loop amplitude responses, $\mathbf{X}(s)=\mathbf{H}(s,\mathbf{X})\mathbf{f}(s)$, are obtained from the harmonic balance using the describing function of the nonlinearities, $\mathbf{N}(s,\mathbf{X})$, where

$$\mathbf{H}(s,\mathbf{X})=\left(\mathbf{M}s^2+\mathbf{C}s+\mathbf{K}+\mathbf{N}(s,\mathbf{X})\right)^{-1} \tag{4}$$

To assign the two eigenvalues (μ_j,μ_j^*), the following equations are solved to provide the control gains.

$$\begin{pmatrix}\mathbf{g}\\ \mathbf{f}\end{pmatrix}=\begin{bmatrix}(\mathbf{H}(\mu_j,\mathbf{X})\mathbf{b})^T & \mu_j(\mathbf{H}(\mu_j,\mathbf{X})\mathbf{b})^T\\ (\mathbf{H}(\mu_j^*,\mathbf{X})\mathbf{b})^T & \mu_j^*(\mathbf{H}(\mu_j^*,\mathbf{X})\mathbf{b})^T\end{bmatrix}^{-1}\begin{pmatrix}-1\\ -1\end{pmatrix} \tag{5}$$

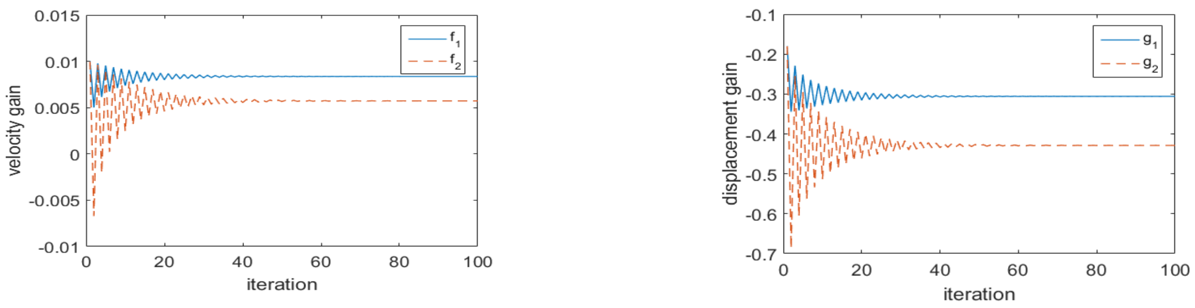
Since, the control force can change the input amplitude, an iterative method is used to obtain the feedback gains. First, an initial amplitude of responses corresponding to the underlying linear system is considered. This amplitude is substituted into Eq.(4) to obtain the open-loop frequency responses. Then, the frequency responses are inserted into Eq. (5) to obtain the initial feedback gains. The stiffness and damping matrices are updated using $\mathbf{K}+\mathbf{b}\mathbf{g}^T$ and $\mathbf{C}+\mathbf{b}\mathbf{f}^T$. The updated matrices are then used in Eq. (4) to obtain the modified amplitude of responses, $\mathbf{X}$. This procedure continues until the feedback gains are converged.

Numerical Simulation

A two DOF system with cubic stiffness nonlinearity is considered as,

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\end{pmatrix}+\begin{bmatrix}0.1 & -0.1\\ -0.1 & 0.1\end{bmatrix}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}+\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0.5x_1^3\\ 0\end{bmatrix}=\begin{bmatrix}f(t)\\ 0\end{bmatrix} \tag{6}$$

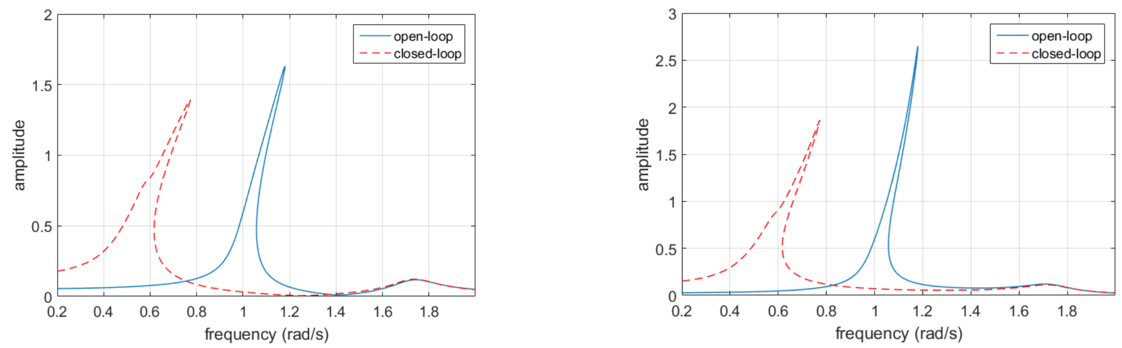
The describing function of the nonlinear force is therefore $\mathbf{N}(s,X_1)=\left[0.75(0.5)X_1^2 \quad 0\right]^T$.The open-loop eigenvalues of the underlying linear system are $\lambda_{1,2}=\pm 1i,\lambda_{3,4}=-0.1\pm 1.72i.$ We wish to assign the eigenvalues of the nonlinear system to $\mu_{1,2} = -0.01 \pm 0.8i$ using the control force distribution $\mathbf{b} = [1 \quad 1]^T$. This vector is orthogonal to the second linear mode, therefore it is expected that the second mode is unaffected particularly at low excitation levels. The iterative procedure is applied to obtain the control gains for example when the forcing amplitude is $F = 0.08N$ as shown in Figure 1(a) and (b).



(a) (b)

Figure 1. The feedback control gains at every iteration (a) velocity gains (b) displacement gains

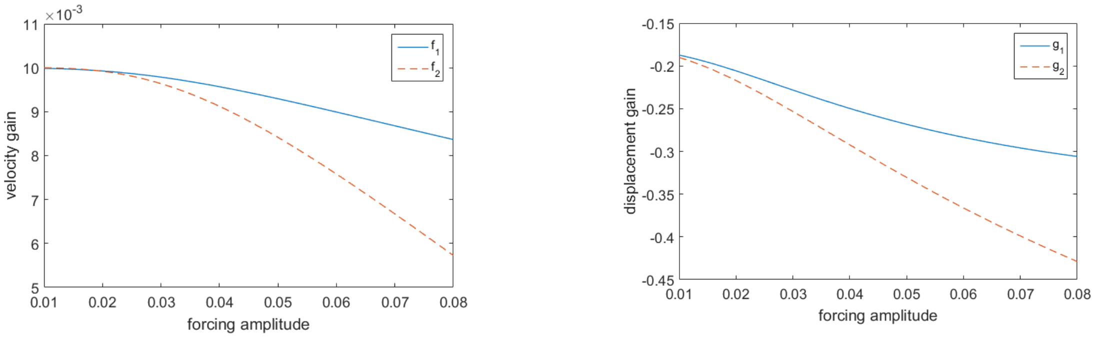
The open-loop and closed-loop amplitude of responses are shown in Figure 2. It can be clearly seen that the peak resonance is assigned at 0.8rad/s. The second mode is unaffected by the control force.



(a) (b)

Figure 2. The amplitudes: open-loop marked with blue solid line and closed-loop marked with red dashed line (a), X_1, (b) X_2

The forcing amplitude is varied from 0.01N to 0.08N and the level-dependent control gains are plotted in Figure 3, which maintains the closed-loop peak resonance at 0.8 rad/s. When the forcing amplitude increases, the nonlinear effects become more evident. For higher forcing amplitudes, the feedback gains do not converge. The convergence of the algorithm will be investigated as part of the future work.



(a) (b)

Figure 3. Level dependent gains at every forcing amplitude (a) velocity gains (b) displacement gains

Conclusions

A control strategy based on the receptances is developed to assign the peak resonance of a MDOF nonlinear system. The method requires an iteration to adjust for the variation of the input amplitude due to the control force. It has been demonstrated that the feedback control gains can assign the peak resonances successfully.

References

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[2] Ghandchi Tehrani, M. Wilmshurst, L and Elliott, S.J. (2013), Receptance Method in Active Vibration Control of a Nonlinear System, *JSV*, 332, 4440-4449.