On the Two Degrees of Freedom Oscillator with Nonlinear Stiffness Coupling: Theoretical and Experiment Results

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Summary. An overview of our main findings on the two degrees of freedom oscillator with coupling nonlinear stiffness is presented. The system is modelled as a linear spring-mass-damper oscillator with an attached nonlinear oscillator consisting of a mass suspended by a parallel combination of a linear damper and a spring with linear and cubic stiffness coefficients. The mass of the linear system is excited by a harmonic force with constant amplitude at each frequency, and the system response is investigated in the assumption that it is predominately harmonic at the frequency of excitation. The analytical expressions of the frequency response curves of the displacements amplitudes are reported, so that the influence of the main parameters can be highlighted. Mechanical prototypes are presented, which are used for experimental validation.

Introduction

The coupling of a nonlinear oscillator to a linear oscillator has been extensively studied in the past literature. Some cases where a nonlinear stiffness coupling has been considered are reported in [1,2]. Evidence of complex phenomena were reported, e.g. periodic, quasi-periodic and chaotic motions, bifurcations and instabilities.
From an engineering point of view, those coupled oscillators have been studied for several applications, e.g. as an energy sink [2], as a vibration absorber [3] or as a vibration neutralizer [4]. Indeed, in the last decade, the improved understanding of the mechanics involved in such a system, has led to a new paradigm shift in dealing with nonlinearity: i.e. its exploitation rather than its avoidance.
The work described here reports the main findings achieved by the authors in the investigation of such a system, since their first paper published in 2009 [5]. The approach followed was predominately motivated and driven by experiments. The mathematical formulation was kept as simple as possible, to be applied by engineers in the fundamental understanding and modelling of experimental phenomena, and for designing engineering systems exploiting nonlinearity to a good use.

Model of the system

The model of the system considered here is illustrated in Figure 1(a) and the corresponding equations of motion are given by

$$\begin{aligned} m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s + c_1\dot{z} + k_1 z + k_3 z^3 &= F\cos(\omega t) \\ m\ddot{x}_s - m\ddot{z} - c_1\dot{z} - k_1 z - k_3 z^3 &= 0 \end{aligned}, \qquad \text{(1a,b)}$$

where $z = x_s - x$ is the relative displacement between the two masses, and the other parameters are specified in Figure 1(a).

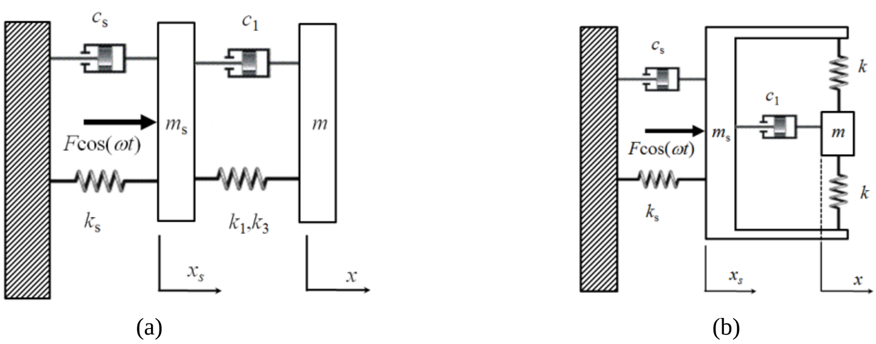

Figure 1. (a) Model of the system under study and (b) its mechanical implementation.

By defining non-dimensional parameters such as the mass ratio, the damping ratios, the frequency ratio and the nonlinearity, Eqs. (1a,b) maybe conveniently rewritten in non-dimensional form and the harmonic balance method can be employed to derive approximate expressions for the amplitude-frequency equations [6].

From an engineering point of view, the model in figure 1(a) may be practically realized by arranging linear springs of stiffness k in a geometric configuration to approximately generate the desired nonlinearity, as depicted in Figure 1(b)

Experimental prototypes

The model of the system depicted in Figure 1 describes the assembly of a nonlinear attachment on the moving head of an electro-dynamic shaker, which also provides the external force excitation. Figures 2(a)-(c) report three experimental rigs, which correspond to the schematic model depicted in Figure 1(b). As an indication, Figures 2(d)-(f) report the frequency response curves associated to the corresponding system in (a)-(c), respectively. In the assumption of harmonic response, the displacement amplitude of the mass m_s is denoted by X_s, and the relative displacement amplitude between the two masses is denoted by Z. In Figures 2(d)-(f), solid/dashed lines denote analytical stable/unstable solutions, and markers denote experimental results.

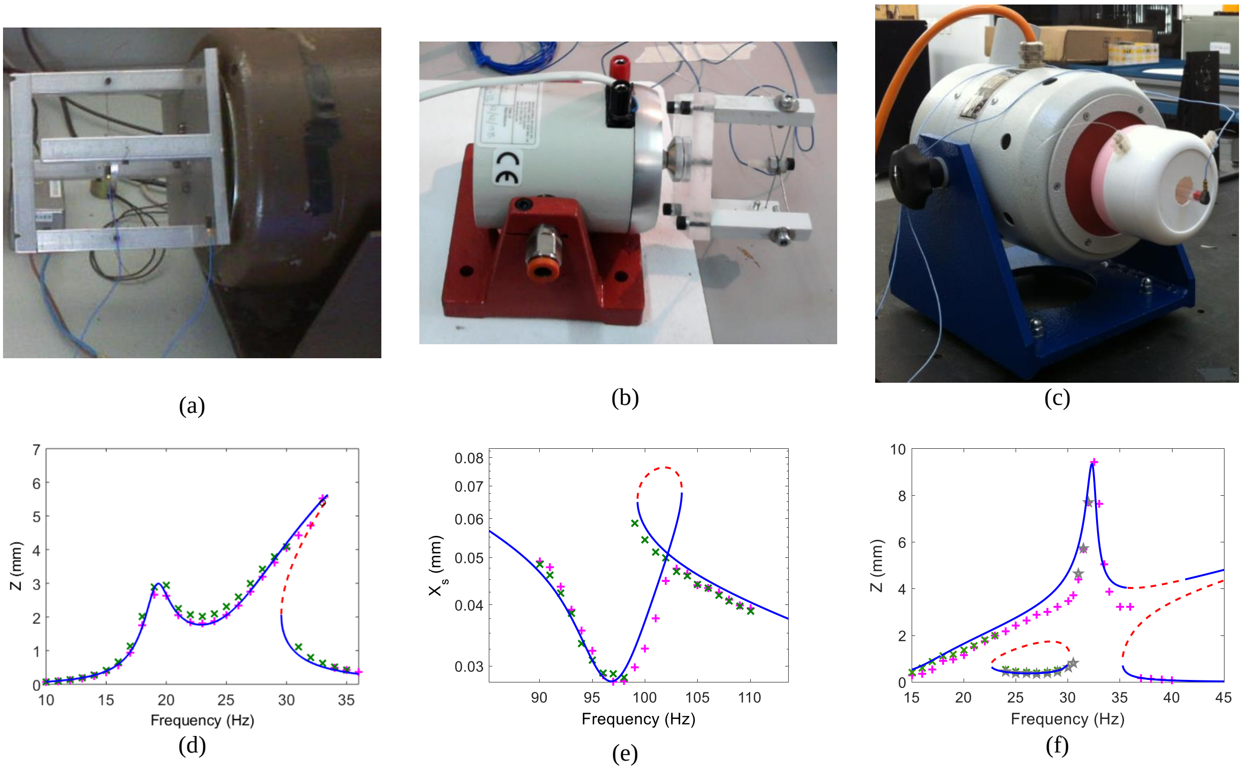

Figure 2. (a)-(c) Experimental rigs, (d)-(f) indicative frequency response curves.

Conclusions

The experimental results reported in this work show a very good agreement with the theoretical expectations. Despite the assumptions adopted and the approximation introduced, the analytical model seems to predict well the fundamental response of the nonlinear system under study. In particular, the experimental detection of a detached resonance curve is achieved [7]. Such an isolated curve appears as a 'bubble' inside the main continuous frequency response curve in Fig. 2(f), as predicted by the authors in [5]. These curves are of particular interest because they could be hidden by numerical or experimental analysis in the case of swept or stepped-sine excitation, and their presence may thus lead to unexpected dramatic changes in the amplitude of the system response.

References

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