Stabilization of a Multi-Tethered Lighter-Than-Air Rigid-body Sphere Undergoing Vortex-Induced Vibrations in Uniform Flow

La Mi* and Oded Gottlieb *,**
*Autonomous Systems and Robotics Program, Technion—Israel Institute of Technology, Haifa, Israel
**Department of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa, Israel

Summary. We consider the stabilization of a lighter-than-air sphere undergoing vortex-induced vibrations in uniform flow. Motivated by recent engineering applications, we extend a single-tether model coupled with a three-dimensional wake oscillator to incorporate a four-tether configuration with a complete rigid-body model for the light sphere. Stabilization of limit-cycle oscillations is achieved for optimal mass ratio and tether length parameters which are obtained by asymptotic analysis of a reduced-order model.

Lab and field experiments have shown that vortex-induced vibrations (VIV) are the cause of self-excited oscillations of tethered light spheres. These limit-cycle oscillations are more pronounced in the direction transverse to the flow [1], exhibiting periodic, quasi-periodic [2] or chaotic-like responses [3]. These observations have been confirmed by a recent theoretical study, where the structural equations were coupled with a three-dimensional wake oscillator [4]. However, the current literature is predominantly focused on single tethered systems, whereas multiple engineering applications employ multi-tether configurations [5]. This research extends the validated model in [4] to the multi-tether framework incorporating the rotational degrees-of-freedom to describe complete rigid-body dynamics.

The dynamical system consists of four equally spaced massless elastic tethers with untensioned length l_0^* connecting to a light sphere (mass ratio $m^* < 1$) at its equator. We assume that the center of gravity, buoyancy and aerodynamic forces coincide with the geometric sphere center and that the incoming flow U_∞ is horizontal in the earth frame, whose origin is located at the static equilibrium sphere center and the x, y, z axes point to the streamwise, transverse and vertical directions respectively as portrayed in the Fig. 1. The equations of motion are formulated in the body frame and then transformed to the earth frame to facilitate the expression of tether restoring forces. The nondimensional equations for the structural state variables $\boldsymbol{\eta} = [x \;\; y \;\; z \;\; \phi \;\; \theta \;\; \psi]'$ and the wake oscillator $\boldsymbol{\nu} = [p \;\; q \;\; r]'$ comprise of an $\mathbb{R}^{18}$ dynamical system which takes the following form:

$$
\begin{aligned}
&\boldsymbol{\eta}_{\tau\tau} + \boldsymbol{\delta}\boldsymbol{\eta}_\tau + \sum_{i=1}^{4} R_i \boldsymbol{q}_{\mathrm{T}i} = \begin{bmatrix}\eta_u u^2(\beta_D + p) & \eta_u u^2 q & \eta_u u^2 r + \eta_g & 0 & 0 & 0\end{bmatrix}' \\
&p_{\tau\tau} + \omega_p^2 p = (\alpha_p - \gamma_p p^2) p_\tau - (k_{p2} q q_\tau + k_{p3} q_\tau^2) + \mu_p x_\tau \\
&q_{\tau\tau} + \omega_q^2 q = (\alpha_q - \gamma_q q^2) q_\tau - (k_{q2} p q_\tau + k_{q3} p_\tau q + k_{q4} p_\tau q_\tau) - (k_{q6} r q_\tau + k_{q7} r_\tau q + k_{q8} r_\tau q_\tau) + \mu_q y_\tau \\
&r_{\tau\tau} + \omega_r^2 r = (\alpha_r - \gamma_r r^2) r_\tau - (k_{r2} q q_\tau + k_{r3} q_\tau^2) + \mu_r z_\tau,
\end{aligned}
\tag{1}
$$

where the term $\sum_{i=1}^{4} R_i \boldsymbol{q}_{\mathrm{T}i}$ is derived from the elastic energy stored in the four tethers; β_D denotes the mean drag coefficients; η_g relates to the contribution of gravity and buoyancy.

Two typical values of the static tether inclination angle β are investigated: $\beta = 60°$ represents an arbitrary configuration and $\beta = 90°$ is a limiting case equivalent to the single-tether model. Numerical integrations for parameters calibrated for the experimental condition in [1] ($m^* = 0.76$, $l_0^* = 8.66$) show that the latter behaves exactly like a single tethered system which does not rotate, whereas the former exhibits large rotational oscillation for high reduced velocity U^*, which is defined by $U^* = U_\infty / f_n d$ where f_n is the structural natural frequency in the transverse direction and d is the sphere diameter. The amplitude responses are depicted in Fig. 2 (left). Additionally, the aperiodic solutions found for $\beta = 60°$ are projected onto the xy plane in Fig. 2 (right).

For the 90° case, the linearization of (1) breaks down into several coupling groups, of which $\{y, q\}$ plays the most

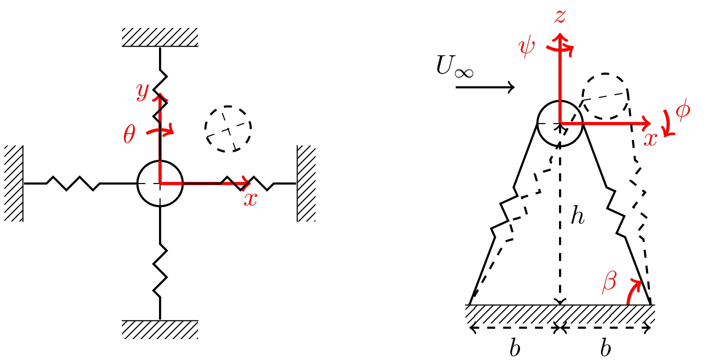

Figure 1: Top view (left) and side view (right) of the multi-tethered light sphere system. The second and fourth tether are not shown in the side view.

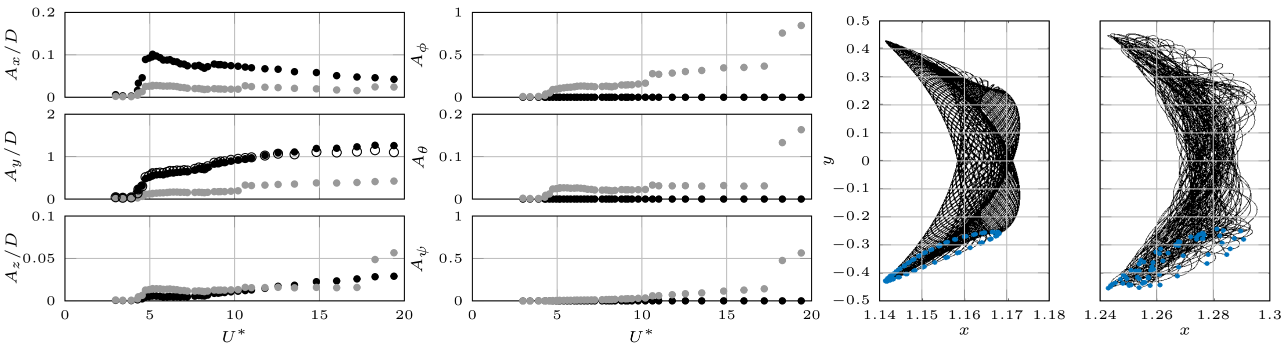

Figure 2: Amplitude responses (left) for $\beta = 60°$ (gray dots) and for $\beta = 90°$ (black dots) compared with the measurements obtained in [1] (black circles). State-space projections $y(x)$ of the quasiperiodic ($U^* = 17.2$) and chaotic-like ($U^* = 18.3$) responses overlaid with Poincare maps for 60° (right).

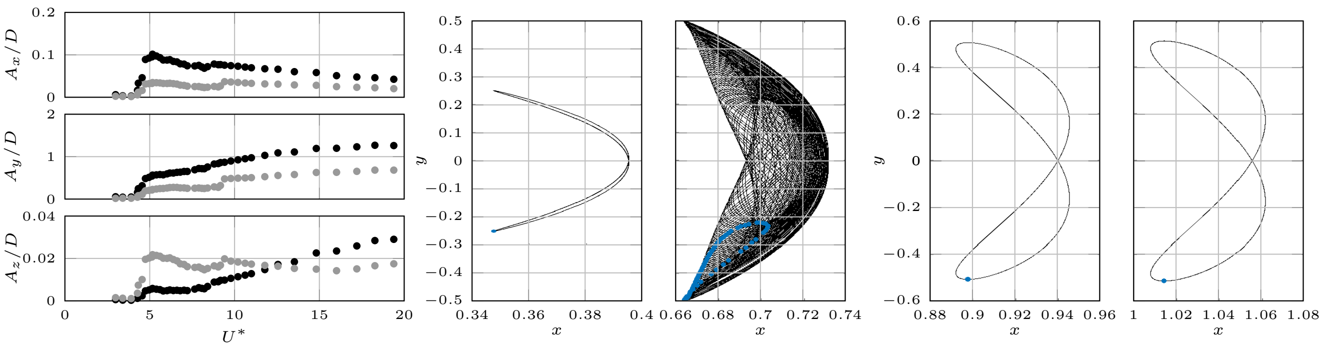

Figure 3: Amplitude responses (left) for $\beta = 90°$ with original parameters (black) compared with new parameters (gray). State-space projections $y(x)$ overlaid with Poincaré maps of periodic ($U^* = 8.1$) and quasi-periodic ($U^* = 10.6$) responses obtained for $\beta = 90°$ with new parameters (middle), and those of regularized periodic responses ($U^* = 17.2$ and $U^* = 18.3$) obtained for $\beta = 60°$ with new parameters (right).

significant role in the transverse fluid-structure interaction. We therefore identify a reduced-order model in $\mathbb{R}^4$:

$$\begin{aligned} y_{\tau\tau} + \omega_y^2 y &= \eta_u u^2 q - \delta_y y_\tau - \tilde{\alpha} y^3 \\ q_{\tau\tau} + \omega_q^2 q &= \left(\alpha_q - \gamma_q q^2\right) q_\tau + \mu_q y_\tau, \end{aligned} \tag{2}$$

where the y equation is truncated to cubic order to facilitate the procedure of multiple scales asymptotic analysis, from which the relationship between the slowly varying amplitude a_y and the system parameters can be determined algebraically [4].

There are two control parameters: the untensioned tether length l_0^* and the mass ratio m^*. Decreasing either one of the control parameters contributes to lowering the slowly-varying transverse amplitude a_y. Note that in practice, one may want to maintain a certain operational height h. This condition dictates reduced tether length l_0^* for decreased mass ratio m^*, both directions contributing to the transverse amplitude suppression. An example of such effect on the full-order system with the parameters $m^* = 0.456$ and $l_0^* = 8.66$ is demonstrated in Fig. 3 (left). However, the solution becomes quasiperiodic beyond $U^* = 9$ as documented in Fig. 3 (right). To ensure orbital stability, additional constraints on the control parameters may be obtained by the stability analysis of the slowly varying evolution amplitudes.

For non-90° cases, following linearization, the y and q components are coupled to ϕ and ψ, which requires an extended form of a reduced-order model in $\mathbb{R}^8$. Nevertheless, we test how reducing m^* and l_0^* subject to constant h as was done for $\beta = 90°$ influences the full-order system for $\beta = 60°$. While the aperiodic solutions obtained in the benchmark 60° simulation are regularized, the limit-cycle amplitudes are increased for the transverse direction, see Fig. 3 (right) compared with Fig. 2 (right). Future research will focus on utilizing the extended reduced-order model to optimize m^* and l_0^* for stabilization of non-90° configurations.

References

[1] Govardhan R. N., Williamson C. H. K. (1997) Vortex-Induced Motions of a Tethered Sphere. *J. Wind Eng Ind Aerod* **69-71**:375-385.

[2] Govardhan R. N., Williamson C. H. K. (2005) Vortex-Induced Vibrations of A Sphere. *J. Fluid Mech* **531**:11-47.

[3] Coulombe-Pontbriand P., Nahon M. (2009) Experimental Testing and Modeling of a Tethered Spherical Aerostat in an Outdoor Environment. *J. Wind Eng Ind Aerod* **97**:208-218.

[4] Mi L., Gottlieb O. (2015) Asymptotic Model-Based Estimation of a Wake Oscillator for a Tethered Sphere in Uniform Flow. *J. Fluid Struct* **54**:361-389.

[5] Vermillion C., Grunnagle T., Lim R., Kolmanovsky I. (2014) Model-Based Plant Design and Hierarchical Control of a Prototype Lighter-Than-Air Wind Energy System, With Experimental Flight Test Results. *IEEE Transactions on Control Systems Technology* **22**:531-542.