Axial and torsional dynamics of a distributed drill string system

Ulf Jakob F. Aarsnes* and Nathan van de Wouw**
* *International Research Institute of Stavanger (IRIS), Oslo, Norway*
DrillWell - Drilling and well centre for improved recovery, Stavanger, Norway
** *Eindhoven University of Technology, Department of Mechanical Engineering, Eindhoven, The Netherlands*
Department of Civil, Environmental & Geo-Engineering, University of Minnesota, Minneapolis, USA
Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands

Summary. This abstract involves the dynamic (stability) analysis of distributed drill-string systems. A minimal set of parameters characterizing the linearized, axial-torsional dynamics of a distributed drill string, including delay model for the bit-rock interaction, is derived, corresponding to eight parameters for a two-sectioned drill-string (e.g., corresponding to the pipe and collar sections of a drilling system). These are used to plot the *inverse gain margin* of the system, parametrized in the non-dimensional parameters, effectively creating a stability map covering the full range of realistic physical parameters. This analysis reveals a complex spectrum of dynamics not evident in analysis with lumped models, thus indicating the importance of analysis using distributed models.

Introduction

The performance of rotary drilling systems used to drill boreholes in the earth is often limited by the occurrence of self-excited vibrations. Self-excited vibrations cause early fatigue of drill pipes and premature failure of bits and, therefore, should be avoided. The underlying cause of the instabilities leading to such self-excited vibrations is the interaction between the drill-string dynamics and a rate-independent bit-rock interaction law through the so-called regenerative effect [3]. This effect introduces a delay based *feedback* in the system, as the current force on the bit, which is a function of the instantaneous depth of cut, depends on the axial position of the bit at an earlier time [2].

Distributed drill-string dynamics model

The linear axial and torsional distributed drill string dynamics are described by the dimensionless irrational transfer functions $g_a(s), g_t(s)$, respectively. These transfer functions can be found from a transfer matrix approach [1], and relates force and torque at the bit (W_b, T_b) to axial and angular velocity at the bit (V_b, Ω_b):

$$\frac{V_b}{W_b}(s) = -\frac{1}{\zeta_a} g_a(s), \qquad \frac{\Omega_b}{T_b}(s) = -\frac{1}{\zeta_t} g_t(s), \tag{1}$$

where ζ_a, ζ_t are the axial torsional pipe impedance, $s \in \mathbb{C}$. The linearized delay-based model for the bit-rock interaction, see Fig. 1, relates weight and torque on bit to the combined depth of cut $D(s)$ [2, 3]:

$$W_b(s) = a\zeta\epsilon D(s), \quad T_b(s) = \frac{1}{2}a^2\epsilon D(s), \qquad D(s) = \frac{N}{s}\Big[V_b(s)\big(1-e^{-t_N s}\big) - R\Omega_b(s)\big(1-e^{-t_N s}\big)\Big], \tag{2}$$

where a, ϵ, ζ, N are parameters dependent on bit and rock properties [3], R is the ratio of the equilibrium rate-of-penetration (ROP), denoted by $\bar{v}$, to the equilibrium angular velocity (RPM) $\bar{\omega}$. Finally, $t_N = \frac{2\pi}{N\bar{\omega}}$ is the (constant) time between two successive cutters passing the same angular position at the equilibrium angular velocity $\bar{\omega}$.

The bit-rock interaction introduces a potentially destabilizing delayed feedback into the system (as depicted in Fig. 2), which can be analyzed by employing the the Nyquist criteria on the characteristic function: $1 + G_a + G_t$, obtained by combining (1) and (2), where:

$$G_a(s) = g_a(s)\frac{K_a}{s}\big(1-e^{-t_N s}\big), \qquad G_t(s) = -g_t(s)\frac{K_t}{s}\big(1-e^{-t_N s}\big), \qquad K_a = \frac{a\zeta\epsilon N}{\zeta_a}, \qquad K_t = \frac{Ra^2\epsilon N}{2\zeta_t}. \tag{3}$$

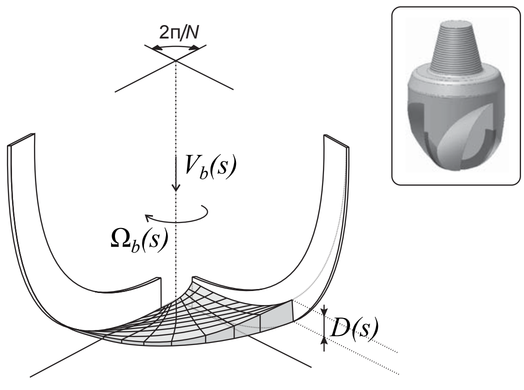

Figure 1: Downhole bit-rock interaction, from [2, 3].

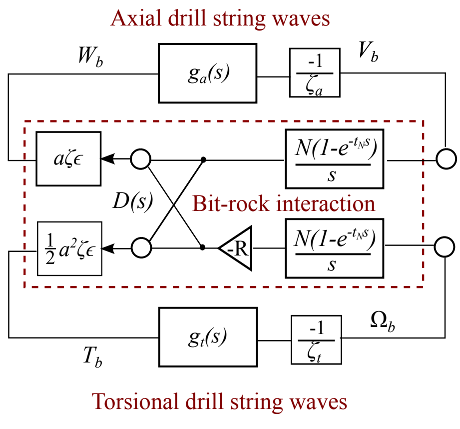

Figure 2: Block diagram of the system.

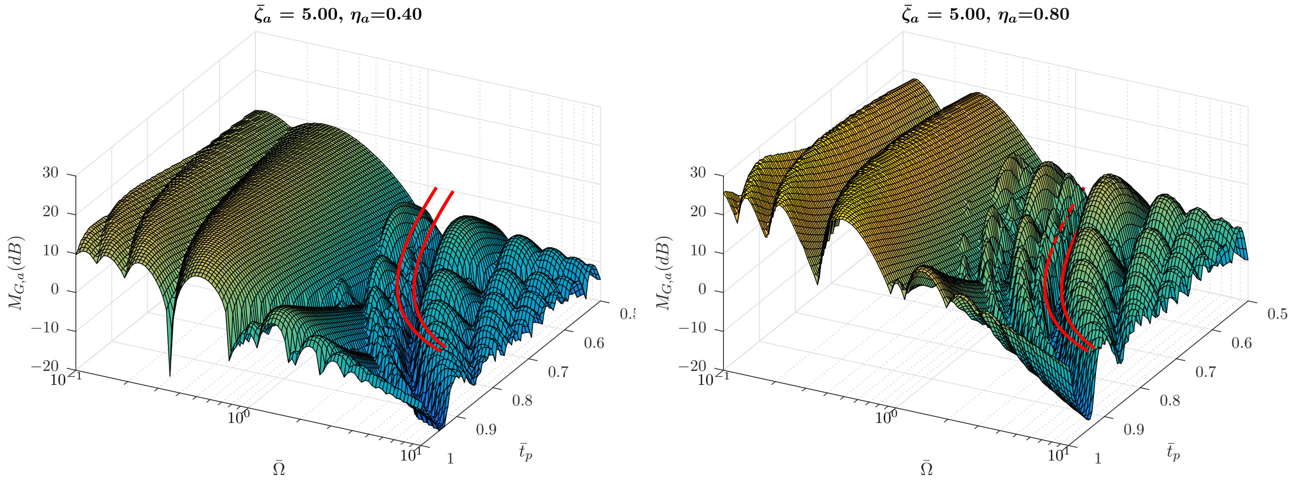

Figure 3: 3D plot of the inverse gain margin, $M_{G,a}$, for a two section pipe, for large amount of damping ($\eta_a = 0.4$, left) and ($\eta_a = 0.8$, right). The stability lobes, characteristic of delay systems, can be clearly seen for $\bar{\Omega}_N < 1$, while for $\bar{\Omega}_N > 1$ the more complex interaction between (anti-) resonances of axial dynamics in $g_a(s)$ and the delay term creates a series of intersecting ridges and troughs, the most pronounced of which is, indicated by red lines.

By introducing the non-dimensional time $\bar{t} = t/t_*$, where $t_* = L/c_t$ is the wave travel time for the torsional wave velocity c_t, we find that the complete two-section drill-string dynamics, with the characteristic equation $1 + G_a + G_t = 0$, can be specified by eight dimensionless parameters: $\bar{\Omega} = \frac{t_* N}{2\pi}\bar{\omega}$ - Dimensionless top-drive RPM relative to the drill string travel time, $\bar{K}_a = K_a t_*, \bar{K}_t = K_t t_*$ - Nominal axial and torsional loop gain coefficients, η_a, η_t - Pseudo reflection coefficients, given by the amount of damping of the axial and torsional dynamics, $\bar{\zeta}_a = \zeta_a^c/\zeta_a^p$, $\bar{\zeta}_t = \zeta_t^c/\zeta_t^p$: The relative size of collar to pipe cross sectional area and polar moment of inertia, respectively, and $\bar{t}_p = t_t^p/t_* \in [0, 1]$: drill pipe (topmost section) travel time relative to total drill string travel time.

Stability analysis results

The derived characteristic parameters can be used to parametrize the stability properties of the system. To this end, we define the normalized inverse of the system gain margin, $M_{G,i},\ i \in \{a, t\}$, as

$$M_{G,i} \equiv \max_{\varpi \in \varpi_{180}} \left| \frac{G_i(j\varpi)}{\bar{K}_i} \right|, \qquad \text{with} \quad \varpi_{180} = \{\varpi \in \mathbb{R} : \angle G_i(j\varpi) = 180\,\text{deg}\}. \tag{4}$$

This means that $\bar{K}_i M_{G,i} > 1$ (i.e., exceeds 0 dB) indicates instability. This analysis reveals complex dynamics for the distributed system, see Fig 3, not evident in analysis obtained using lumped models. The main trends that emerge are that decreases in the inverse gain margin (leading to a more stable system) is obtained by:

1. Decreasing the reflection coefficient, c.f. left and right side of Fig. 3.
2. Increasing the angular top-drive velocity, $\bar{\Omega}$. The magnitude of the peaks can be seen to decrease for increasing $\bar{\Omega}$ in Fig. 3.

The presented result also enables a more structured approach to analyzing the non-local (non-linear) dynamics through the derivation of the minimal set of characteristic parameters.

References

[1] U. J. F. Aarsnes and O. M. Aamo, "Linear stability analysis of self-excited vibrations in drilling using an infinite dimensional model," *Journal of Sound and Vibration*, vol. 360, pp. 239–259, jan 2016. [Online]. Available: http://linkinghub.elsevier.com/retrieve/pii/S0022460X15007385

[2] C. Germay, V. Denoël, and E. Detournay, "Multiple mode analysis of the self-excited vibrations of rotary drilling systems," *Journal of Sound and Vibration*, vol. 325, no. 1-2, pp. 362–381, aug 2009. [Online]. Available: http://linkinghub.elsevier.com/retrieve/pii/S0022460X09002478

[3] T. Richard, C. Germay, and E. Detournay, "A simplified model to explore the root cause of stickslip vibrations in drilling systems with drag bits," *Journal of Sound and Vibration*, vol. 305, no. 3, pp. 432–456, aug 2007. [Online]. Available: http://linkinghub.elsevier.com/retrieve/pii/S0022460X07002908