Steady Streaming in A Vibrating Channel with Ratchet

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Summary. An experiment demonstrated that in a narrow channel between two parallel plates lined with asymmetrical sawteeth, fluid could be pumped along channel from one end to the other when the plates transversally vibrate against each other. We here put forward a theory to describe this ratcheting effect of fluid, by developing a 'terrain-following' boundary-layer approach. Analytical solutions are obtained. The pumping action is due to the steady momentum flux that arises from the nonlinear convective inertia of the oscillating flow and is maintained by viscous dissipation. The theoretical results qualitatively agree well with the observations.

Extended Abstract

Motions rectified by symmetry-breaking mechanisms in oscillating flows have been of great interest in biological locomotion and engineering applications. Recently, an experiment (Thiria and Zhang 2015) demonstrated that in a narrow channel between two parallel plates, with their facing sides lined with asymmetrical sawteeth and harmonically oscillating oppositely and normally, fluid can be pumped from one end to the other. Since the directional transport is achieved without valves, this demonstration of fluid ratcheting using geometric asymmetry also offers an alternative idea for valveless pumps, which remain active interests in microfluidics and biomedical engineering applications. Inspired by the experiment, we put forward here a theory describing the ratcheting effect of fluid (Yu 2014). We solve the vorticity equation and boundary conditions, as follows.

$$\frac{\partial}{\partial t}\nabla^2\psi - \alpha\beta\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial\left(x,y\right)} = \frac{1}{2}\sigma^2\beta^2\nabla^2\nabla^2\psi, \quad 0 < y < y_{\rm s}(x) + \alpha\beta\cos t, \tag{1}$$

$$\psi = 0, \quad \partial^2\psi/\partial y^2 = 0, \quad \text{at} \quad y = 0 \quad \text{(channel centerline)}, \tag{2}$$

$$\partial\psi/\partial y = 0, \quad \partial\psi/\partial x = \sin t, \quad \text{at} \quad y = y_{\rm s}(x) + \alpha\beta\cos t \quad \text{(channel wall)}. \tag{3}$$

where $\partial(\,,)/\partial(x,y)$ is the Jacobian determinant for nonlinear convection terms and $y_s(x)$ is the sawtooth wall profile of wavelength λ. The parameters are $\alpha = a/h_0$, $\beta = kh_0$, $\sigma = \delta/h_0$, where $\delta = \sqrt{2\nu/\omega}$ is the Stokes boundary layer thickness, a and ω are the amplitude and angular frequency of wall vibration, ν is the kinematic viscosity, $k = 2\pi/\lambda$ and h_0 is the mean half channel width. A 'terrain-following' boundary-layer approach is formulated using a conformal transformation (Yu and Howard 2012). In the mapped plane, we invoke a boundary layer approximation and seek the perturbation solution $\psi = \psi_0 + \alpha\psi_1 + \cdots$, where $\alpha \ll 1$ is necessary to avoid the collision of sawteeth. The analytical solutions are given for the induced steady flow field and net pumping rate of fluid. The rectification of time harmonic motion is seen due to (i) the steady momentum flux that arises from the nonlinear convective inertia of the oscillating flow, and (ii) the interaction of vorticity with the wall motion, that arises due to the fluid velocity near the wall being out of phase with the wall velocity, as a result of viscosity. The geometric asymmetry renders these effects to be spatially biased, leading to a unidirectional component in the steady flow.

The theoretical findings qualitatively agree well with the experimental observations, e.g. the pumping rate increasing with the forcing frequency and amplitude (consistent with the fact that the phenomenon is due to the inertial effects), the left-shift of the stagnation point during the closing half cycle when the fluid exits the channel. Effects on the net pumping rate have been examined varying parameters representing oscillation conditions and sawtooth geometry. We have also clarified the effects of entrance and exit flow conditions due to the geometries at channel ends. This is a secondary source of spatial asymmetry in the system, and can cause a net directional pumping even when the sawteeth are individually symmetrical. Whereas the influences from both sources of symmetry-breaking can be comparable in a short channel with a few sawteeth, the accumulative effects of asymmetric wall profile become dominant as the number of sawteeth increases, determining the net pumping (rate and direction) of fluid in a long channel.

This work is a demonstration of judicious use of the conformal transformation method, which was developed in the research on ocean waves (Yu and Howard 2012), to study the vorticity dynamics. Here, a boundary-layer approximation is invoked for the sake of obtaining an analytical solution. This can be relaxed if numerical solutions are sought. The mapping functions can also be adapted to be time-dependent to consider large-amplitude wall oscillations, providing an alternative in numerical modeling of flows with complex boundary geometry.

References

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[3] YU, J. 2014 Fluid ratcheting by oscillating channel walls with sawteeth. *J. Fluid Mech.* **761**, 305–328. doi:10.1017/jfm.2014.636

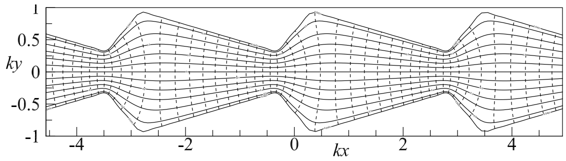

Figure 1: The curvilinear grids in the (x, y) plane that correspond to the Cartesian grids $\xi = const$ and $\eta = const$ in the mapped plane, showing the 'terrain-following' contours near the walls.

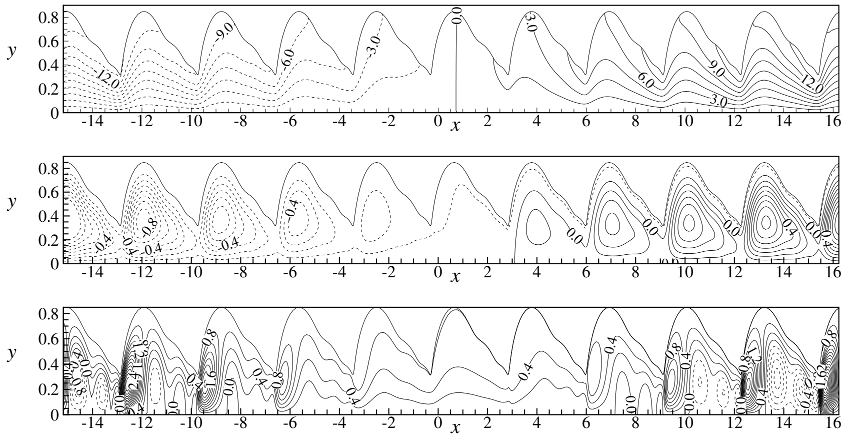

Figure 2: Contours of stream functions in the half plane $y > 0$. (a) ψ_0 at $t = \pi/2$. $-15.9312 \leq \psi_0 \leq 15.4848$. (b) ψ_0 at $t = \pi$, i.e. at the end of a closing half cycle when the flow is about to reverse. $-1.0654 \leq \psi_0 \leq 0.8901$. (c) The steady streaming flow field ψ_{10}, showing the net pumping from left to tight. $-0.8882 \leq \psi_{10} \leq 3.4980$. Dashed lines indicate the negative contour values. The Reynolds number, based on the speed of vibration and channel width, is $Re = 2.47$ taking water as the working fluid.

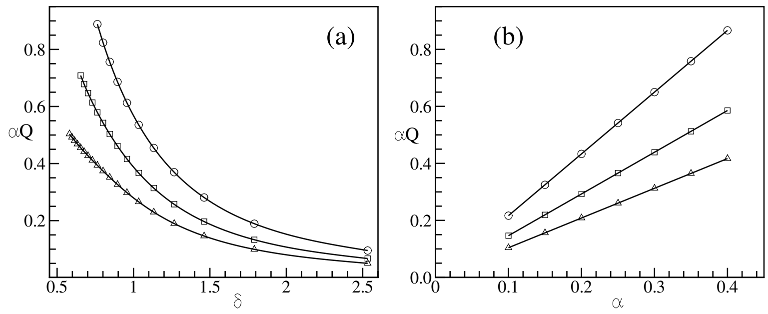

Figure 3: The pumping rate αQ (for half channel width) (a) as a function of frequency (via the dimensionless Stokes layer thickness $\delta = \sqrt{2\nu/\omega}/h_0$) for oscillation amplitude $\alpha = 0.3$; (b) as a function of α for $\delta = 0.9247$. Channel lengths: $\bigcirc$, $N = 15$; $\square$, $N = 10$; $\triangle$, $N = 5$ (multiple of λ_s). $\epsilon_b = 1.0$, $\beta = 0.3130$.

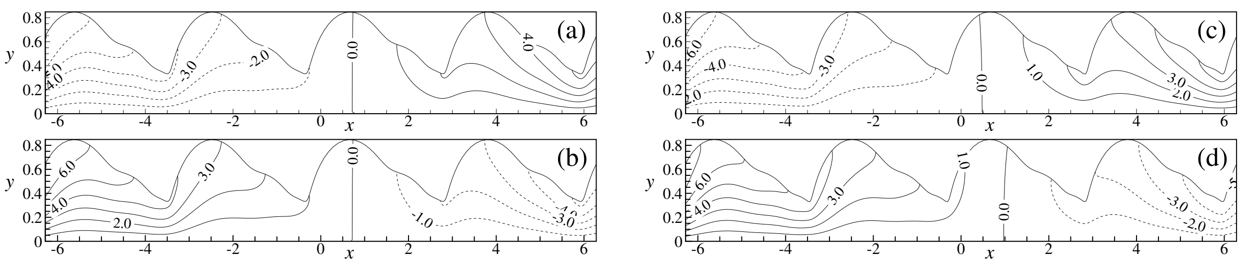

Figure 4: The leading order (linear dynamics) stream function ψ_0 (a) during the closing half cycle and (b) during the opening half cycle, showing the complete reversal of the flow pattern as expected for low Reynolds number flow of an incompressible Newtonian fluid. The streaming function $\psi_0 + \psi_{10}$ including the steady streaming (c) during the closing phase and (d) during the opening phase, showing the shift of stagnation point that is consistent with the observation (Thiria and Zhang 2015).