Nonlinear Vibration Energy Harvesting Using Piezoelectric Tiles Placed in Stairways

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Summary. This article investigates the use of piezoelectric tiles in stairways for vibration energy harvesting - harnessing electrical power from naturally occurring vibrational phenomena - from pedestrian traffic. While harvesting using such tiles in flat walkways have been studied and implemented, the greater amount of kinetic energy naturally required to traverse stairways suggests potentially better prospects for harvesting. This, and a set of allied questions are studied using linear, nonlinear, and linearly coupled nonlinear analytical models for two types of commercially available piezoelectric tiles. The models comprise a system of two coupled ODEs representing the mechanical and electrical degrees of freedom. The influence of a stochastic model of pedestrian traffic on the results is also studied.

Introduction

Current global environmental concerns demand the identification and utilization of previously unrealized sources of energy. One such source is that of ambient kinetic energy, often manifest in the form of random vibrations. Previously unharnessed, it is now feasible to convert such energy to usable power through the use of vibration energy harvesters (VEH). Realizable on both macro and nano scales, VEH offer great potential for autonomous power generation. Much of the early work on VEH focused on the powering of wireless devices, particularly sensors [1, 2, 3, 4, 5, 6, 7], while more recent reviews can be found in [8, 9]. The potential benefits of nonlinear harvesters is also an area of ongoing research [10, 11, 12, 13, 14, 15]. In this nonlinear regime, harvesters with bistable potentials have been of particular interest [16, 17, 18, 19]. The ambient excitation driving VEH is characteristically random, and several investigations have taken this into consideration [20, 21, 22].

Along with theoretical analyses, the real world applications of VEH continue to expand. Among the more exciting developments has been the consideration of VEH tiles. These tiles have been deployed in public walkways to harvest the kinetic energy of pedestrian traffic. For instance, tiles installed in railway stations in Japan have successfully powered ticket gates [23]. A club in London, UK was also able to meet 60% of its energy requirements using tiles installed within its dance floor [23]. The efficacy of VEH tiles was also the subject of a study at an Australian university [24].

The objective of this article is to investigate using analytical and computational methods, the application of VEH tiles in stairways. Specifically, the effort aims to: (1) Compare the power output from VEH-tiles in stairways to that of tiles on flat walkways, (2) investigate the effects of using tiles with nonlinear stiffness on the power output, (3) study the effects of linear coupling between tiles in adjacent steps, (4) compare the outputs from two types of commercially available tiles and (5) study the influence of randomness in pedestrian traffic (modeled as an additive White Gaussian noise component in the forcing function) on the power output. The motivation behind the work is that pedestrians must exert more work when climbing up stairs than when walking across flat ground. The effort is carried out in the following steps. First, a system model for a linear harvester is presented. The model is then modified to include nonlinear stiffness terms and linear coupling between adjacent tiles. Numerical simulations are run using the model, all subjected to both flat ground and stairway forcing scenarios. The effects of considering additive White Gaussian noise in the excitation of the tiles is also investigated.

Development of a System Model

Much work has been done on the subject of modeling piezoelectric harvesters, see for example [25]. For this investigation, we have adopted a compressive piezoelectric harvester model from [26]. The resulting equations are included as (1) and (2).

$$m\ddot{x} + c\dot{x} + kx + Nv = f(t) \tag{1}$$

$$-N\dot{x} + C\dot{v} + v/R = 0 \tag{2}$$

In these equations, x represents the change in thickness of the harvester, while v represents the developed voltage. The constants m, c, C, and k represent the mass, damping coefficient, capacitance, and stiffness of the piezoelectric harvester, respectively. N is the electromechanical coupling coefficient, which relates the applied force to the produced voltage. N is defined in equation (3). The piezoelectric coefficient g_{33} relates the applied stress to the developed voltage for this peizo material. A is the cross sectional area of the harvester, and h_o is the thickness when not strained.

$$N = \frac{A}{g_{33}h_o} \tag{3}$$

The system equations were non-dimensionalized using $x = x_c y$, $t = t_c\tau$, and $v = v_c\nu$. The non-dimensionalized equations are presented in (4) and (5).

$$\ddot{y} + \frac{c}{\sqrt{km}}\dot{y} + y + N^2\nu = F(\tau) \tag{4}$$

$$-\frac{1}{Ck}\dot{y} + \dot{\nu} + \frac{1}{CR}\sqrt{\frac{m}{k}}\nu = 0 \tag{5}$$

It is important to note that equations (1) and (2) (and their non-dimensionalized counterparts (4) and (5)) represent the dynamics of a single harvester. In reality, several harvesters would be deployed on each step in a staircase or tile in a walkway. Since the dynamics considered at this point is in the linear regime, it is logical to combine each harvester's differential equation into a single equation for analysis. The stiffness of an array of piezo harvesters assembled on a step may then be obtained by considering them as a set of parallel springs. Hence, an equivalent stiffness coefficient can be found by summing the individual stiffness coefficients, k. The equivalent mass and capacitance of the system of piezo harvesters can be found in a similar manner. As for the number of harvesters, it was decided that four would likely be used to support each step on a staircase or tile in a walkway. Consequently the equivalent stiffness, capacitance, and mass are simply four times their values for a single tile.

The forcing function, $f(t)$, represents the input from pedestrian footfalls. A footfall can be well represented as a momentary pulse, before and after which the exerted force on the ground is zero. Representing such a pulse is accomplished using a Dirac delta function multiplied by the magnitude of the footfall force. This representation can then serve directly as the forcing function for the piezoelectric tile model.

Determining the magnitude of the aforementioned pulse both when walking on flat ground and up stairways offers another challenge. Fortunately, results offering insight on human induced loading of staircases are available in the literature [27]. These results found the most natural walking pace up a flight of stairs to be 2Hz, with a footfall magnitude of about 42% of a pedestrian's body weight. For walking on flat ground, the most comfortable frequency is 1.95 Hz with a magnitude of about 35% of the pedestrian's body weight. This frequency was taken as 2 Hz in order to simplify the implementation of the Dirac function. A pedestrian mass of 80.7 kg was used. This was the average mass of an adult in the U.S. in 2012 [28]. The acceleration due to gravity was taken as 9.81 m/s^2.

For all simulations, the effects of addition of noise to the forcing signal was investigated. Specifically, white Gaussian noise was added to the magnitude of the footfall force. This was intended to represent the variation between footfall forces for the same "average" 80.7 kg person.

Also of interest is the performance of a system of harvesters with a nonlinear stiffness. This system is represented in equations (6) and (7). In these equations, α is the coefficient of the quadratic term in the potential , while β is the coefficient of the quartic term in the potential corresponding to the Duffing equation. Considering a system of four harvesters per step or tile as in the linear case is more complicated here. The stiffness coefficients cannot be simply added due to the nonlinearity of the system. It was decided to start with the equivalent linear system of a four-harvester step and adapt it to the nonlinear case. On an empirical basis, the coefficients α and β were taken as fractions of the equivalent stiffness of four linear harvesters in parallel. Several different combinations of different α and β values were tested, as described in the results section.

$$m\ddot{x} + c\dot{x} + \alpha x + \beta x^3 + Nv = f(t) \tag{6}$$

$$-N\dot{x} + C\dot{v} + v/R = 0 \tag{7}$$

These system equations were also nondimensionalized for numerical simulations, with the result in (8) and (9).

$$\ddot{y} + \frac{c}{\sqrt{\alpha m}}\dot{y} + y + \frac{\beta}{\alpha^3}y^3 + N^2\nu = F(\tau) \tag{8}$$

$$-\frac{1}{C\alpha}\dot{y} + \dot{\nu} + \frac{1}{CR}\sqrt{\frac{m}{\alpha}}\nu = 0 \tag{9}$$

Lastly, a linearly coupled system of four nonlinear steps was studied, guided by a model available in the literature [10]. The modeling equations for a single step in this four step setup are included below as equations (10) and (11). In these equations, the subscript i indicates the tile while $i-1$ and $i+1$ indicate its neighbors. The coupling stiffness is assumed to be equal to α.

$$m\ddot{x}_i + c\dot{x}_i + \alpha x_i + \beta x_i^3 + Nv_i + k_{cpl}(2x_i - x_{i-1} - x_{i+1}) = f(t) \tag{10}$$

$$-N\dot{x}_i + C\dot{v}_i + V_i/R = 0 \tag{11}$$

The non-dimensionalized form of these equations are obtained as (12) and (13).

$$\ddot{y}_i + \frac{c}{\sqrt{\alpha m}}\dot{y}_i + y_i + \frac{\beta}{\alpha^3}y_i^3 + N^2\nu_i + \frac{k_{cpl}}{\alpha}(2y_i - y_{i-1} - y_{i+1}) = F(\tau) \tag{12}$$

$$-\frac{1}{C\alpha}\dot{y}_i + \dot{\nu}_i + \frac{1}{CR}\sqrt{\frac{m}{\alpha}}\nu_i = 0 \tag{13}$$

The coefficient values used for the models in this paper are included in table 1. Values for these coefficients were taken from a commercially available 45x45x5mm harvester that utilizes the Navy Type V piezoelectric material [29]. The value of the damping coefficient c was not disclosed by the manufacturer. Consequently it had to be estimated using the natural frequency of the linear unforced system. It was found to be approximately equal to $1e^5$.

Table 1: Model Coefficients

$m = 0.3195kg$	$c = 1e^5 kg/s$	$k = 2.16e^{11} kg/s^2$
$g_{33} = 24.2e^{-3} Vm/N$	$C = 3.28e^{-8} F$	$A = 2.025e^{-3} m^2$
	$h_o = 5e^{-3} m$	

Results

Numerical simulations were performed for each of the models under parameter variations. The main goal was to determine if tiles performed better under stairway forcing than under flat ground forcing. Also of interest was the potential benefits of nonlinearity and the effect of noise in the magnitude of the footfall force. The first simulations performed were for a single linear step. A nonlinear step was then considered, followed by the aforementioned setup of four, linearly-coupled, nonlinear steps. Lastly an alternative piezoelectric material was considered.

Numerical simulations were ran using the "ode45" function in MATLAB. These simulations produced a tile's voltage output as function of time. This was in turn used to calculate the energy delivered to a resistive load over a ten second period. As the load resistance was not known, a sensitivity analysis with a broad range of resistances was performed. When pertinent, the MATLAB "awgn" funtion was used to add noise to the forcing signal.

The first single step simulations ran utilized a linear stiffness. Stairway as well as flat ground forcing were applied to this step. The addition of white noise was also investigated. The results of these simulations are depicted in figure 1. As apparent in this figure, the load resistance plays a strong role in the magnitude of the power output. Under a 5 Ω load, it would at first seem apparent that the step performed best under a flat ground forcing scenario. However, deeper investigation into the underlying voltage displacement results revealed erratic system behavior. Not until the load resistance was increased to above 500 Ω did the step produce smooth, predictable voltage displacements. Lastly, it can be seen in the results for this linear system with a 500 Ω or greater load that the differences between the two forcing scenarios is negligible. The addition of noise also has virtually no effect, regardless of the signal-to-noise ratio used in the simulations.

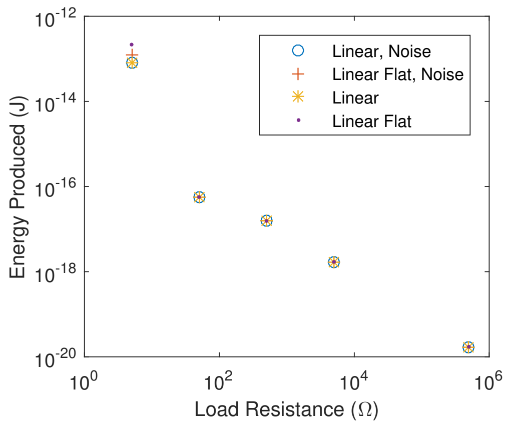
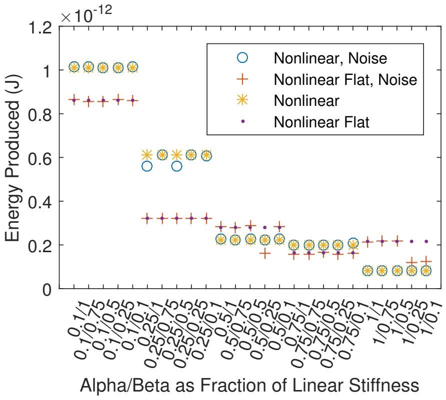
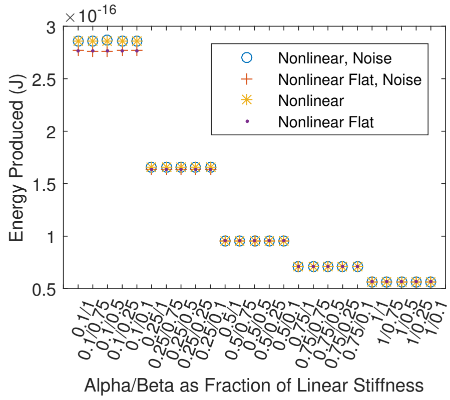


Figure 1: Linear System Energy Output

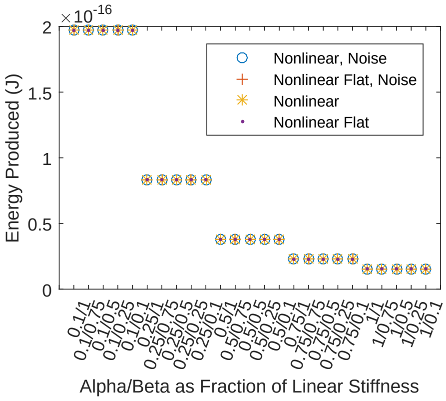
Next the nonlinear system was investigated. As mentioned previously, no nonlinear version of the harvester is available commercially. Hence the magnitudes of the stiffness coefficients had to be assumed. As such, all nonlinear simulations performed included sensitivity analyses that explored different combinations of stiffness coefficients α and β. Just as for the linear system, simulations were performed under both flat ground and stairway forcing, with and without the addition of noise. These results are depicted in figures 2a through 2d. The x-axis in these results graphs represent the different combinations of the nonlinear stiffness coefficients α and β, calculated as fractions of the original linear stiffness. Those with smaller α values represent more strongly nonlinear systems. As evident in the results, smaller α values correspond to higher system outputs. This demonstrates that strongly nonlinear systems out perform weakly nonlinear systems. Also evident is the fact that while α is held constant and only β is varied, the output is essentially left unchanged. Consequently it can be concluded that variation in the β component of the nonlinear stiffness has negligible effect. Figure 2a depicts the same unreliability in results under low resistive loads that the linear simulations exhibited. Just as for the linear case, results became much more reliable around the 500 Ω mark. In this more reliable region of system performance, it is again apparent that stairway versus flat ground, and noise versus no noise exhibit no real difference in output.



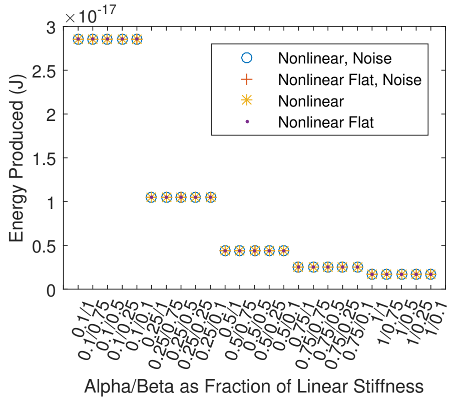
(a) 5 Ω Load



(b) 50 Ω Load



(c) 500 Ω Load



(d) 5 kΩ Load

Figure 2: Nonlinear Results

Knowing that a low α stiffness coefficient produced the best nonlinear response, attention naturally turned to comparing the output of this nonlinear system to that of the linear one considered earlier. These results are depicted in figure 3. It can be seen that the linear results coincide with those of the weakly nonlinear system. This is not particularly surprising as increasing the value of α essentially brings the nonlinear, non-dimensionalized system closer to a linear one. This graph also demonstrates a clear advantage of using a strongly nonlinear harvester as the energy produced is several times greater.

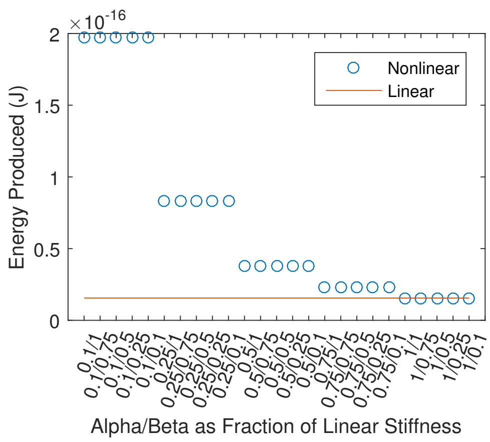
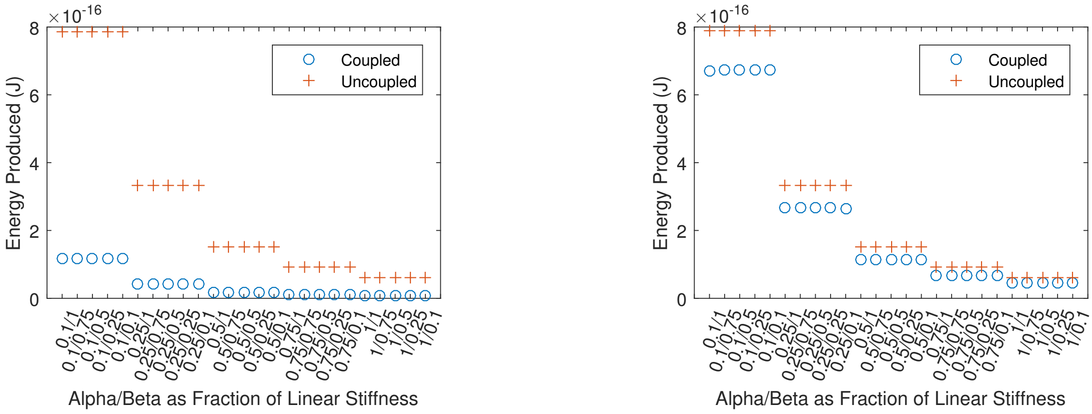


Figure 3: Linear vs. Nonlinear, R = 500Ω

With the nonlinear step system established as the most desirable, the potential benefits of linear coupling between four of



(a) Cascading Steps, 500Ω

(b) Simultaneous Steps, 500Ω

Figure 4: Coupled vs. Uncoupled

these steps was explored. Using the model outlined previously, simulations were performed under two different forcing scenarios. The first scenario involved a series of cascaded steps. This represents a single person stepping on one step after the other, making his or her way up the stairs. A second person does not embark up the steps until the first person has completed his or her journey. The second forcing scenario simply forces each step simultaneously at a rate of 2 Hz. Both of these forcing regimes were tested under stairway loading only as the objective was to determine the benefit of coupling, not flat versus stairway placement. The results of both of these forcing scenarios are depicted in figures 4a and 4b. These figures demonstrate that in both of these scenarios, coupling between the steps is actually detrimental to the power output. This likely due to steps acting as motion inhibitors to their neighbors. The addition of noise had no effect on this conclusion.

The final set of simulations explored the performance of an alternative piezoelectric material. All simulations previously used the aforementioned Navy Type V piezoelectric material. Using the same harvester device dimensions, a step using harvesters of the Navy Type III piezoelectric material was simulated. This material has been explored previously in a compressive, piezoelectric harvesting scenario [26] and was thus a natural choice. Coefficient values were taken from tables provided by the manufacturer of the harvester considered previously [30]. Figure 5 demonstrates that the Type V material outperforms the Type III material. This is likely due to the fact that the Type III material has a larger elastic modulus. Consequently, it displaces less and in turn produces smaller voltage outputs.

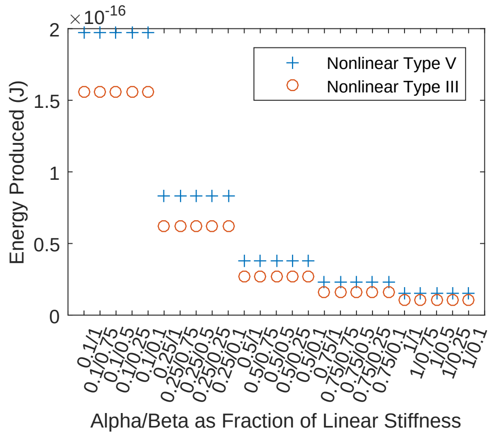


Figure 5: Type III vs. Type V, 500Ω

Conclusion

This paper studied the application of piezoelectric tiles placed on stairways subjected to human traffic. Based upon existing literature, modeling equations were derived for both a linear and a nonlinear step. A system of linearly coupled, nonlinear steps was also developed. The outputs of the linear and nonlinear systems under stairway forcing, flat ground forcing, with and without noise for several load resistances were all compared. No significant difference in output was found between stairway and flat ground forcing. However, strongly nonlinear systems outperformed their linear counterparts and this suggests that the design and fabrication of nonlinear piezoelectric tiles ultimately leading to commercial availability could potentially have a huge impact on the future of vibration energy harvesting. Among the currently available tiles, Navy Type V was observed to outperform Navy Type III. Low load resistances resulted in erratic system behavior, while higher loads resulted in reduced output. The addition of additive white noise had no significant effect, while linear coupling

between nonlinear steps was found to be detrimental.
In the light of the results reported in this paper, certain aspects invite further investigation. Firstly, the pedestrian footfall representation could be improved by invoking a Poisson arrival rate of traffic. Secondly, it would be interesting to compare the results with those of simulations based upon a standard numerical scheme for stochastic differential equations such as the Euler-Maruyama method. The authors intend to pursue these directions in future work.

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