Delay system modelling and analysis of a down-hole tool in drilling systems

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Summary. In this abstract, we present a delay systems model for the coupled axial-torsional dynamics of a drilling system including a down-hole tool. The down-hole tool directly affects the coupling between the axial and torsional dynamics and aims to improve the rate-of-penetration of the drilling system, thereby improving drilling efficiency. The results presented here provide physics-based insight in the working principle of the tool and confirm that indeed drilling efficiency can be improved by its usage.

Introduction

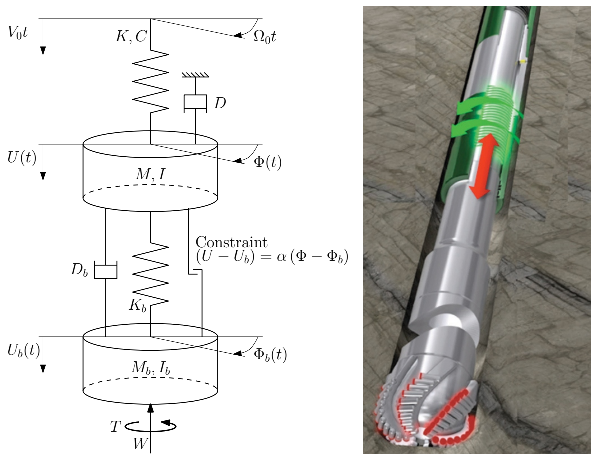
Drilling systems are used for the harvesting and exploration of oil, gas, mineral and geo-thermal energy resources. The rate-of-penetration of the drilling system is a key factor in the total drilling costs. In particular, for geo-thermal applications reducing drilling costs increases the feasibility of the realization of wells for geo-thermal applications. In this abstract, we investigate the effect of a down-hole tool [1], called the anti-stall tool, on the rate-of-penetration of a drilling system. Given the fact that the tool directly affects the coupling between the axial and torsional motion of the drilling system, we adopt a drill-string model incorporating both axial and torsional modes of vibration [2, 3]. Below, the model is presented, which consists of a model of the drill-string dynamics, a model of the tool and a bit-rock interaction model introducing state-dependent delays in the dynamics. Subsequently, the analysis of the resulting rate of penetration is conducted.

Delay model for the drill-string dynamics including the down-hole tool

A schematic representation of the drill-string model including the tool is shown in Figure 1(a). Herein, the drilling system is separated in two parts: above and below the tool, which is located in the bottom-hole assembly (BHA). The part above the tool is modelled as a discrete mass M with inertia I and represents the first modal inertia of the combined drill-string and the top part of the BHA. The bottom part represents the part of the BHA below the tool and is modelled as a mass M_b with inertia I_b. An impression of the anti-stall tool (AST) is given in Figure 1(b). It consists of two tool bodies connected to each other with a helical spline and an internal pre-loaded spring. The principle of the tool is that a torsional load with sufficient magnitude to overcome the loading in the compressed spring will make the upper tool part with internal helical spline rotate onto the mating lower part. When the upper and lower parts screw together in this manner, the tool telescopically contracts and the drill-string becomes shorter. The generalized coordinates describing the torsional and axial displacement of the system are given by $q = \begin{bmatrix} U & U_b & \varPhi & \varPhi_b \end{bmatrix}^T$, with U the axial position of the BHA (above the AST), $\varPhi$ the angular position of the BHA (above the AST), U_b the axial position of the bit (i.e. below the tool) and $\varPhi_b$ the angular position of the bit. Following [3], the forces $W = W^c + W^f$ and torques $T = T^c + T^f$ on the bit are modelled as

$$W^c = na\zeta\varepsilon d, \quad W^f \in nal\bar{\sigma}\frac{1+\mathrm{Sign}\left(\dot{U}_b\right)}{2}, T^c = \frac{1}{2}na^2\varepsilon d, \quad T^f \in \frac{1}{2}na^2\xi\mu l\bar{\sigma}\frac{1+\mathrm{Sign}\left(\dot{U}_b\right)}{2}, \tag{1}$$

where $\mathrm{Sign}(\cdot)$ denotes the set-valued sign function. The parameters in (1) reflect the properties of the bit and the rock [3]. The depth-of-cut d is determined by $d(t) = U_b(t) - U_b(t - t_n(t))$. The delay t_n itself is time-dependent (actually state-dependent) and denotes the time interval in which the bit rotates $2\pi/n$ rad, which is the angle between two successive



(a) (b)

Figure 1: (a) Schematic model of a drill-string including anti-stall tool (AST). (b) Impression of the working principle of the AST [4].

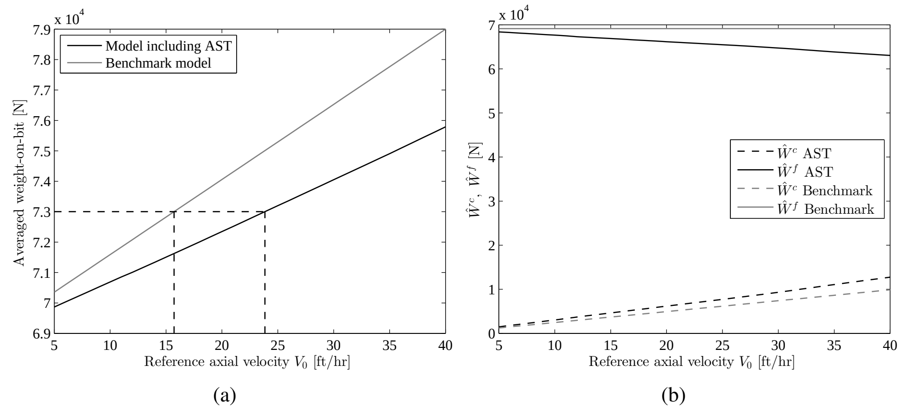


Figure 2: (a) Averaged value $\hat{W}$ of the total weight-on-bit W and (b) averaged value of the cutting force $\hat{W}^c$ and wearflat contact force $\hat{W}^f$, both as function of the prescribed top-drive axial velocity V_0, for $\Omega_0 = 120$ rpm.

blades of cutters on the bit: $\Phi_b(t) - \Phi_b(t - t_n(t)) = \frac{2\pi}{n}$. To model the AST, an additional axial spring K_b and axial damper D_b are introduced and a kinematic constraint is introduced that describes the coupling between the axial and torsional displacement, due to the helical spline in the tool. The related holonomic constraint equation follows from the relation between the lead p, lead angle β and the pitch radius r of the tool and is given by $U - U_b = \frac{p}{2\pi}(\Phi - \Phi_b)$. The top drive boundary conditions are given by an imposed angular velocity Ω_0 and an imposed axial velocity V_0.
A Lagrangian approach for systems with constraints is used to derive the equations of motion for this system, resulting in the equations of motion given by:

$$\begin{aligned}
&M\ddot{U} + D\dot{U} + D_b\left(\dot{U} - \dot{U}_b\right) + K\left(U - V_0 t\right) + K_b\left(U - U_b\right) = -\lambda \\
&M_b\ddot{U}_b - D_b\left(\dot{U} - \dot{U}_b\right) - K_b\left(U - U_b\right) = -W^c - W^f + \lambda \\
&I\ddot{\Phi} + C\left(\Phi - \Omega_0 t\right) = \tfrac{p}{2\pi}\lambda \\
&I_b\ddot{\Phi}_b = -T^c - T^f - \tfrac{p}{2\pi}\lambda
\end{aligned} \tag{2}$$

with the kinematic constraint described above. Herein, λ is the associated Lagrange multiplier. After elimination of the holonomic constraint, the comprised model, in independent coordinates, can be described in terms of a delay differential inclusion with state-dependent delay.

Impact of the tool on the rate-of-penetration

The results of a numerical simulation case study are depicted in Figure 2(a). Herein, the *averaged* weight-on-bit $\hat{W}$ is depicted for a range of axial top-drive velocities V_0 and an angular top-drive velocity of $\Omega_0 = 120$ rpm (other parameter settings are according to [5]). In this figure, also the results for a benchmark model without the tool, see [2], are displayed. These results show that for a given rate-of-penetration V_0, the averaged weight-on-bit is significantly reduced by using the tool. In other words, the same rate-of-penetration V_0 can be achieved with a lower average weight-on-bit, which indicates the fact that the tool improves drilling efficiency. An alternative interpretation of these results is that, for an equal average weight-on-bit, a higher rate-of-penetration can be attained. An explanation for this improvement in drilling efficiency can be found in Figure 2(b), which shows that the tool, firstly, decreases the wearflat contribution $\hat{W}^f$ of the averaged weight-on-bit by the tool retraction and, secondly, increases the cutting contribution $\hat{W}^c$ of the averaged weight-on-bit, where this cutting contribution is in fact responsible for generating rate-of-penetration. The latter effect provides a physics-based explanation for the improvement in drilling efficiency by the employment of the tool.

Conclusions

A delay systems model for a drilling system including an anti-stall tool is developed. Dynamic analyses using this model show that the tool can improve the drilling efficiency in terms of the rate-of-penetration of the drilling system.

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