Direct antiresonance continuation for non linear dynamic absorbers

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Summary. We address non linear torsional vibration absorbers (TVA), used in rotating machinery to counteract irregularities of rotation at some order of the engine speed of rotation. The TVA is analogous to a tuned mass damper (TMD), tuned on the desired order. It exhibits non-linearities of various natures which affect resonance and antiresonance frequencies at large amplitude of motion, which consequently cause the detuning of the system from the targetted order. We propose an original direct antiresonance continuation method based on a numerical path following method. This procedure allows an accurate and fast prediction of the operating point of the system as a function of the amplitude of oscillation of the TVA.

Introduction

Non linear torsional vibration absorbers (TVA) are used in rotating machinery to counteract irregularities of rotation, called "acyclisms", at a some order of the engine speed of rotation. These passive devices have the ability to adapt to the engine speed of rotation and remain tuned on a given order. They operate as TMD with variable stiffness. In practice, the TVA consists to an oscillating mass subjected to move in a particular path [5] on a primary inertia which is linked to the rotating assembly. However the TVA includes strong non-linearities: geometric non-linearities and those due to Coriolis effect, both intrinsic to rotating articulated systems. They cause the detuning of the TVA and the shifting of the operating order of the TVA (an antiresonance of the whole system) from the targeted engine order [7]. Knowledge of the operating point behavior of the device is therefore essential to ensure optimal acyclisms filtering.
This study focuses on the computation of the periodic solutions via a path following method, the asymptotic numerical method (ANM) [2], coupled to the harmonic balance method (HBM) [3]. The direct antiresonance continuation procedure proposed here is adapted from a classical limit point tracking method [6]. The limit point is detected via the computation of a test function which adds a constraint to the initial system to solve and allows the continuation with two free parameters instead of only one, in our case the frequency and the amplitude of the external forcing.

Periodic solutions

The equations of motion of the TVA can be written in general following form :

$$\boldsymbol{M}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{f_{in}}(\boldsymbol{x},\dot{\boldsymbol{x}}) + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{f_{int}}(\boldsymbol{x}) = \boldsymbol{F}\cos(\omega t)\,, \tag{1}$$

where $\boldsymbol{x}$ is the vector of unknowns. $\boldsymbol{M}(\boldsymbol{x})$ is the mass matrix and depends on $\boldsymbol{x}$. $\boldsymbol{C}$ is the damping matrix. $\boldsymbol{f_{in}}(\boldsymbol{x},\dot{\boldsymbol{x}})$ is the inertial forces vector, including Coriolis terms, and depends on $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$. $\boldsymbol{f_{int}}(\boldsymbol{x})$ is the internal forces vector and depends on $\boldsymbol{x}$. Here, because only one oscillator is forced, the external forces vector is $\boldsymbol{F} = [0 \ \cdots \ f \ \cdots \ 0]$ where f is the amplitude of excitation of the forced oscillator. Assuming a periodic solution of (1), the vector of unknowns is expanded in a truncated Fourier series :

$$\boldsymbol{x}(t) = \boldsymbol{x}^{(0)} + \sum_{h=0}^{H} \boldsymbol{x}^{(\mathrm{c}h)}\cos(h\omega t) + \boldsymbol{x}^{(\mathrm{s}h)}\sin(h\omega t). \tag{2}$$

Then substituting (2) into (1) and applying HBM, we obtain a system of algebraic equations relating $\boldsymbol{x_0}$, $\boldsymbol{x_{ci}}$, $\boldsymbol{x_{si}}$, ω and f. The accuracy of the periodic solution depends on H, the number of harmonics retains on (2). The final system to solve can be written :

$$\boldsymbol{R}(\boldsymbol{U},\omega,f) = 0 \tag{3}$$

where $\boldsymbol{U} = \left[\boldsymbol{x}^{(0)} \ldots \ \boldsymbol{x}^{(\mathrm{c}h)} \ \boldsymbol{x}^{(\mathrm{s}h)} \ldots \ \boldsymbol{x}^{(\mathrm{c}H)} \ \boldsymbol{x}^{(\mathrm{s}H)}\right]^{\mathrm{T}}$. Finally, (3) is solved by an asymptotic numerical method, for which a quadratic recast of $\boldsymbol{R}(\boldsymbol{U},\omega,f)$ is convenient [3]. In practice, the software Manlab 2.0 is used [1]. Unlike to predictor-corrector algorithms, where the solution is computed point by point, ANM adopts a piecewise continuous representation of the solution using a power series expansion of the pseudo arc length along the branch of solution.
Classical continuation procedure consist in imposing the value of ω or f and keeping the other one free, in order to obtain continuation with respect to f or ω in forced vibration [3].

Antiresonance continuation

The procedure consists in following a point of antiresonance, defined as a minimum of a response amplitude, denoted by z, with respect to the forcing frequency ω. We then consider an augmented non linear algebraic system of the following form :

$$\tilde{\boldsymbol{R}}(\tilde{\boldsymbol{U}},\omega,f) = \left\{ \begin{array}{l} \boldsymbol{R}(\boldsymbol{U},\omega,f) = \boldsymbol{0} \\ z - f(\boldsymbol{U}) = 0 \end{array} \right. \tag{4}$$

where $\tilde{\boldsymbol{U}} = [\boldsymbol{U}\ z]^T$, z is an additional scalar unknown accounting for a particular amplitude. If the h-th. harmonics of the i-th component of $\boldsymbol{x}$ is under concern, z and f read $z = \sqrt{x_i^{(ch)2} + x_i^{(sh)2}}$.
A limit point is a particular point F of the resonance curve for which the tangent is vertical (Fig. 1(a)), i.e. $\partial \boldsymbol{U}/\partial\omega$ tends to infinity. The idea consists in rewriting (4) so that z is a free parameter instead of ω:

$$\boldsymbol{R}^*(\boldsymbol{U}^*, z, f) = \tilde{\boldsymbol{R}}(\tilde{\boldsymbol{U}}, \omega, f) = 0 \tag{5}$$

where $\boldsymbol{U}^* = [\boldsymbol{U}\ \omega]^T$. In the same way than for a limit point, in an antiresonance point, the Jacobian matrix $\boldsymbol{J}^*$ of $\boldsymbol{R}^*$ with respect to $\boldsymbol{U}^*$ becomes singular at the antiresonance. Indeed, differentiating (5) leads to:

$$\frac{\partial \boldsymbol{R}^*}{\partial \boldsymbol{U}^*}\frac{\mathrm{d}\boldsymbol{U}^*}{\mathrm{d}z} + \frac{\partial \boldsymbol{R}^*}{\partial z} = 0 \tag{6}$$

At the antiresonance point A (Fig. 1(a)), the slope of $\partial z/\partial\omega$ is zero, $\partial\omega/\partial z$ is infinite, so that $\partial \boldsymbol{U}^*/\partial z$ becomes undefined (since ω is a component of $\boldsymbol{U}^*$). Since $\partial \boldsymbol{R}^*/\partial z$ is non-null [8], $\boldsymbol{J}^* = \partial \boldsymbol{R}^*/\partial \boldsymbol{U}^*$ must be singular. This condition could be achieved by equating to zero the determinant of $\boldsymbol{J^*}$. In practice, we use the eigenvector $\boldsymbol{\Phi}$ of $\boldsymbol{J}^*$ [4], such that:

$$\det(\boldsymbol{J}^*) = 0 \quad \Leftrightarrow \quad \exists \boldsymbol{\Phi} \neq 0;\ \boldsymbol{J}^*\boldsymbol{\Phi} = 0. \tag{7}$$

The final system to solve then reads:

$$\begin{cases} \tilde{\boldsymbol{R}}(\tilde{\boldsymbol{U}}, \omega, f) = 0 \\ \boldsymbol{J}^*\boldsymbol{\Phi} = 0 \\ ||\boldsymbol{\Phi}|| - 1 = 0 \end{cases} \tag{8}$$

The third equation of (8) normalizes $\boldsymbol{\Phi}$ to ensure that $\boldsymbol{\Phi} \neq 0$ and the uniqueness of the solution.

Application

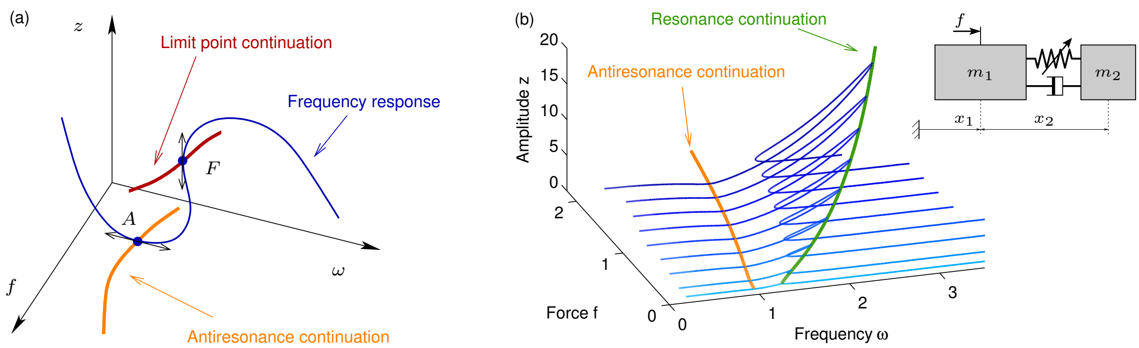

Figure 1: (a) Geometric illustration of limit point and antiresonance continuation; (b) nonlinear tuned mass damper and frequency response of the first harmonics of x_1 for several value of the forcing f and antiresonance and resonance continuation.

Here, we consider a free-free non linear tuned mass damper (Fig. 1(b)) whose equations of motion can be written :

$$\begin{cases} \ddot{x}_1(m_1 + m_2) + \ddot{x}_2 m_2 = f\cos(\omega t) & \text{(9a)} \\ \ddot{x}_1 m_2 + \ddot{x}_2 m_2 + c\dot{x}_2 + kx_2 + \gamma x_2^3 = 0 & \text{(9b)} \end{cases}$$

where c, γ and k are the damping, non linear and linear stiffness constants, respectively. The procedure proposed in this article allows to follow the minimum or the maximum amplitude of the dynamic response as a function of the frequency and the amplitude of the external forcing, as shown on figure 1. This paper follows and extends an earlier antiresonance continuation of undamped system procedure [7].

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