Locomotion conditions for a two-body system on a rough inclined plane

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Summary. The motion of a two-body system along a straight line on an inclined (in particular, horizontal) plane is considered. Anisotropic dry friction is assumed to act between each of the bodies and the plane of motion. The motion of the system is excited by changing periodically the distance between the bodies. Necessary and sufficient conditions for the motion of the system upward along the inclined plane or along a horizontal plane are obtained for the case where the force of friction is small in comparison with the excitation force.

Introduction

The system under consideration can be regarded as a simple model of limbless worm-like locomotors. This study continues the investigations of [1-3]. In [1], the optimal parameters that maximize the average velocity of a similar system moving along a horizontal straight line are found; the distance between the bodies is assumed to change periodically. In [2], the conditions subject to which the system can move in a prescribed direction are found for a special kind of control in the case of small friction; the average velocity of the motion is defined. The motion of the system along a rough isotropic horizontal plane is investigated (mostly numerically) in [3]. In our study, we obtain the conditions for the rectilinear motion of the system along a rough anisotropic inclined plane for an arbitrary periodic law of changing the distance between the bodies, provided that friction is small.

Mechanical model

A mechanical system that consists of two bodies of masses M_1 and M_2 is moving along a plane inclined at an angle of α Let x_1 and x_2 be the coordinates of the bodies measured along the line of motion of the system. The motion is excited and controlled by changing the distance l between the bodies in accordance with a prescribed T-periodic law $l(t) = x_2 - x_1$ (Figure 1).

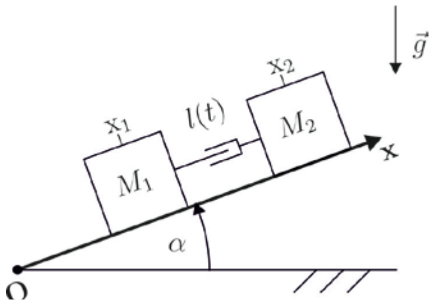


Figure 1. Two-body system on a rough inclined plane

Let the dry friction force defined by the relations $F_i(z) = k_- M_i g \cos\alpha$, for $z < 0$ and $F_i(z) = -k_+ M_i g \cos\alpha$, for $z > 0$, $-k_+ M_i g \cos\alpha \le F_i(z) \le k_- M_i g \cos\alpha$ for $z = 0$ ($i = 1, 2$) act on the respective bodies. According to the linear momentum principle, the motion of the center of mass of the system under consideration is governed by the equation that in dimensionless variables $t = t^* / T$, $x_i = x_i^* / L$, $F_i = F_i^* / F_0$ (the asterisk denotes the dimensional variables) has the form

$$\dot{V} = \varepsilon G(V,t), \qquad G(V,t) = -\tan\alpha / k_{\max} + m_1 F_1(V - m_2 u(t)) + m_2 F_2(V + m_1 u(t)),$$
$$\varepsilon = F_0 T^2 / (ML), \quad m_i = M_i / M \ (i = 1, 2), \quad M = M_1 + M_2, \quad u(t) = \dot{l}(t). \tag{1}$$

Here, $V = m_1 \dot{x}_1 + m_2 \dot{x}_2$ is the velocity of the center of mass of the system, L is a parameter that has a dimension of length, $F_0 = k_{\max} Mg \cos\alpha$, $k_{\max} = \max(k_-, k_+)$.
We assume that $\varepsilon << 1$ and average the first equation of (1) with respect to the explicit time t over the period. Thus we obtain the averaged equation

$$\dot{V} = \varepsilon \overline{G}(V), \qquad \overline{G}(V) = -\tan\alpha / k_{\max} + \int_0^1 \big(m_1 F(V - m_2 u(t)) + m_2 F(V + m_1 u(t))\big) dt \,. \tag{2}$$

Basic results

We are interested in whether there exists a steady-state solution $V_0 \neq 0$ of Eq. (2). If such a solution exists we will say that the locomotion of the system is possible. For an inclined plane, of most interest is the motion upward ($V_0 > 0$), and we will consider only such motions. The necessary and sufficient conditions for the motion upward along an inclined plane are given by

$$k_-(m_1\tau_+ + m_2\tau_-) > k_+(m_1\tau_- + m_2\tau_+), \quad 0 < \alpha < \arctan\big(k_-(m_1\tau_+ + m_2\tau_-) - k_+(m_1\tau_- + m_2\tau_+)\big). \tag{3}$$

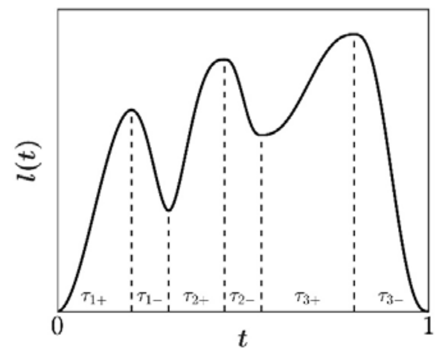
Here, τ_+ is the total time during which $u(t) > 0$ and τ_- is the total time during which $u(t) < 0$, $\tau_+ + \tau_- = 1$ (Figure 2). Thus, the upward motion along an inclined plane is possible even for the case where the coefficient of friction resisting the upward motion (k_+) exceeds the coefficient of friction resisting the downward motion (k_-).
For the case of a horizontal plane, the conditions for the motion of the center of mass become ($V_0 \neq 0$)

$$k_-(m_1\tau_+ + m_2\tau_-) \neq k_+(m_1\tau_- + m_2\tau_+)\,. \tag{4}$$

For the case of isotropic friction, relations (3) and (4) imply, respectively,

$$(m_1 - m_2)(\tau_+ - \tau_-) > 0\,, \qquad (m_1 - m_2)(\tau_+ - \tau_-) \neq 0\,. \tag{5}$$

Thus, locomotion along an isotropic rough horizontal line is possible if and only if the bodies have different masses and, in addition, the total times during which the bodies are moving closer to and apart from each other are different.



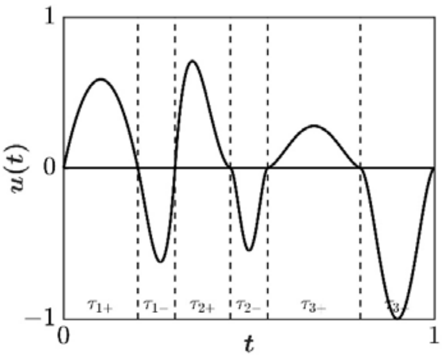


Figure 2. Functions $l(t)$ and $u(t) = \dot{l}(t)$; $\tau_+ \neq \tau_-$

Conclusions

For any periodic continuous piecewise differentiable function that defines the distance between the bodies, the motion along a rough horizontal plane or the upward motion along an inclined plane with constant coefficient of friction are possible if and only if (1) the bodies have different masses and (2) the total times during which the distance between the bodies increases and decreases are not equal to each other. Prototypes of the locomotion system under consideration are built. The experiments with these prototypes confirm the theoretical results.

Acknowledgements

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