Dynamics in Milling Pocket Structures

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Summary: The dynamics in milling thin-walled pocket structures is studied. The multi-pocket structures are regarded as a combination of Kirchhoff plates by using the subdomain decomposition method. The state-dependent cutting zone is considered in formulating the theoretical model. The structural responses during the machining process are simulated by the developed model. We preliminarily studied the different features in thin wall milling compared with the traditional milling dynamics models.

Introduction

Dynamics in milling process have received considerable attention from the researchers, and the issues in thin wall milling are more difficult due to the inherent complexity characterized by the low stiffness, multi-mode coupling, and the low radical immersions. Pocket structure is a kind of thin-walled workpiece which is typically used in the aerospace industry[1]. Insperger *et al.*[2] proposed the generic semi-discretization method for the stability analysis of delayed system. Long *et al.* [3] used the semi-discretization method to study the stability in milling process. Balachandran and Zhao developed a dynamic model to cover the nonlinear effects in milling, but actually only consider one mode in each direction[4]. The literatures which may concerning the dynamics in thin wall milling mainly transplanted the traditional model to the thin wall milling, where the case is regarded as a common milling operation only with a low radical immersion. In this paper, we try to develop a model to cover the dynamic characteristics in thin wall milling. And the preliminarily simulation results are presented.

Theoretical Modelling

A typical pocket structures are shown in Fig. 1, which can be seen as a combination of plates.

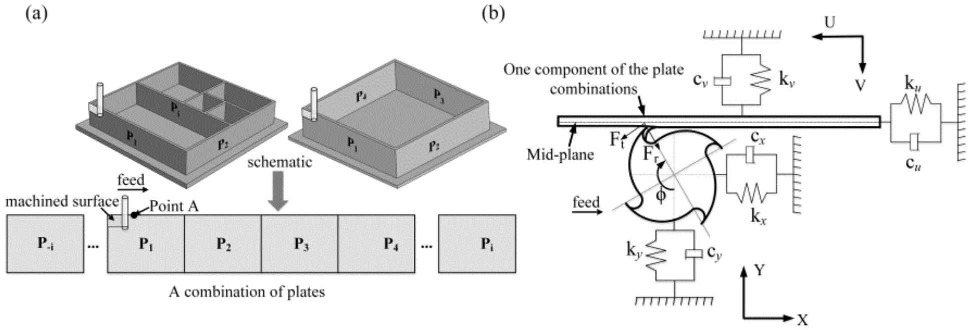

Fig. 1 pocket milling (a) a combination of plates (b) the milling force

For the plate dynamics based on the Kirchhoff hypothesis, the governing equation for this milling process can be set up as

$$\begin{aligned} & m_x\ddot{q}_x + c_x\dot{q}_x + k_x q_x = F_x(t,\tau) \\ & m_y\ddot{q}_y + c_y\dot{q}_y + k_y q_y = F_y(t,\tau) \\ & m_u\ddot{q}_u + c_u\dot{q}_u + k_u q_u = F_u(t,\tau) \\ & D\left(\frac{\partial^4 q_v}{\partial\hat{x}^4} + 2\frac{\partial^4 q_v}{\partial\hat{x}^2\partial\hat{y}^2} + \frac{\partial^4 q_v}{\partial\hat{y}^4}\right) + \rho h\frac{\partial^2 q_v}{\partial t^2} = F_v(t,\tau) + f(\hat{x},\hat{y},\dot{q}_v) \end{aligned} \tag{1}$$

where $m_i, c_i, k_i \ (i = x, y, u)$ are the modal mass, damping, and stiffness in the corresponding direction. And F_x, F_y, F_u and F_v is the milling force in each direction. The last equation describes the motion of plate along the transverse direction with the excitation of cutting force.

The pocket structure can be seen as a combination of plates with the appropriate boundary conditions. According to the subdomain decomposition method presented in our former efforts [5], each plate component may be further divided into a series of small subdomains, and the continuity conditions between the plate components or the small subdomains in one plate component confirm that the vibration displacement and the slope along the interface must be continuous, and this requires that $q_{v,i} = q_{v,i+1}$, $\partial q_{v,i}/\partial n = \partial q_{v,i+1}/\partial n$. In case of the clamped edge, the displacement and slope are zeros. Along the interface between the plate components, the displacement is zero and the slope is continuous. Then, the governing equation may be expressed in the reduced generalized coordinates. Noting that the proposed equation actually considered only one mode in the *x*, *y*, and *u* directions, and this model may be further generalized to incorporate the higher order modes in the other directions. Besides, the perfection of this model depends on the perfection of the milling force model.

Vibration of this pocket structures is actually the boundary value problem of the partial differential equation (PDE) based on the Kirchhoff hypothesis, the weak form for this boundary value problem is

$$\Pi = \int_{t_0}^{t_1} \sum_i \left(T_i - U_i + W_i\right) dt + \int_{t_0}^{t_1} \sum_{i,i+1} \Pi_{\lambda\kappa} dt \tag{2}$$

Then discretized equation for vibration of the pocket structure can be obtained by setting the generalized energy function to be zero, namely $\delta\Pi = 0$. And the governing equation for the pocket milling process can thus be obtained as

$$\begin{aligned}
&m_x\ddot{q}_x + c_x\dot{q}_x + k_x q_x = F_x\left(t,\tau\right)\\
&m_y\ddot{q}_y + c_y\dot{q}_y + k_y q_y = F_y\left(t,\tau\right)\\
&m_u\ddot{q}_u + c_u\dot{q}_u + k_u q_u = F_u\left(t,\tau\right)\\
&m_{v1}\ddot{q}_{v1} + c_{v1}\dot{q}_{v1} + k_{v1} q_{v1} = \Phi_1\left(\hat{x},\hat{y}\right) F_v\left(t,\tau\right)\\
&m_{v2}\ddot{q}_{v2} + c_{v2}\dot{q}_{v2} + k_{v2} q_{v2} = \Phi_2\left(\hat{x},\hat{y}\right) F_v\left(t,\tau\right)\\
&\cdots\\
&m_{vn}\ddot{q}_{vn} + c_{vn}\dot{q}_{vn} + k_{vn} q_{vn} = \Phi_n\left(\hat{x},\hat{y}\right) F_v\left(t,\tau\right)
\end{aligned} \tag{3}$$

The semi-analytical expression for the modes of pocket structures makes the integration of this governing equation simpler. Assuming the workpiece holds the same mid-plane with different thickness during the machining process, the proposed model can also cover the material removal effects.

By considering the state-dependent cutting zone, the milling force can be given as

$$\begin{Bmatrix} F_x\left(t;\tau\right) \\ F_y\left(t;\tau\right) \end{Bmatrix} = \begin{bmatrix} \kappa_{11}\left(t\right) & \kappa_{12}\left(t\right) \\ \kappa_{21}\left(t\right) & \kappa_{22}\left(t\right) \end{bmatrix} \begin{Bmatrix} q_x(t) - q_x(t-\tau) + q_u(t) - q_u(t-\tau) + lf\tau \\ q_y(t) - q_y(t-\tau) + q_v(t) - q_v(t-\tau) \end{Bmatrix} \tag{4}$$

then the solution of the governing equation with a state dependent nonlinear cutting force can be got by using the 4th order runge-kutta method.

Results and Discussions

To investigate the responses in milling the pocket structure, time domain simulations are carried out to predicted the vibration depicted as Fig. 1. The predicted vibration of a given point A at the free edge of the pocket is shown in Fig. 2. The numerical results in Fig. 2(a) reveal that the maximum vibration displacement of point A occurs at nearly 6s, which is close to the position when the cutter moves across point A. Different from the traditional model, the vibration displacement of the pocket is the superposition of each mode. Fig. 2(b) display the relative ratio of the first 7 modes and the physical displacement compared with the first mode, results show that the higher modes may have a larger contribution to the physical displacement, this makes thin wall milling different from the traditional models. Especially for the closed shape structures where the higher modes may be close to each other, for example the pocket structures in this paper and the circular shells. Fig. 2(c) is the FFT spectrum of the time domain simulation at 6s, besides the traditional harmonics of tooth passing frequencies, there is a slight chatter frequency component, this can also be seen in Fig. 2(a). However, as indicated in the numerical simulation, this kind of slight chatter can be ignored since the cutter pass the point A in a short time and then the chatter frequency only exists in a very short period of time. This dynamic feature makes the thin wall milling again different from the traditional quasi-static models.

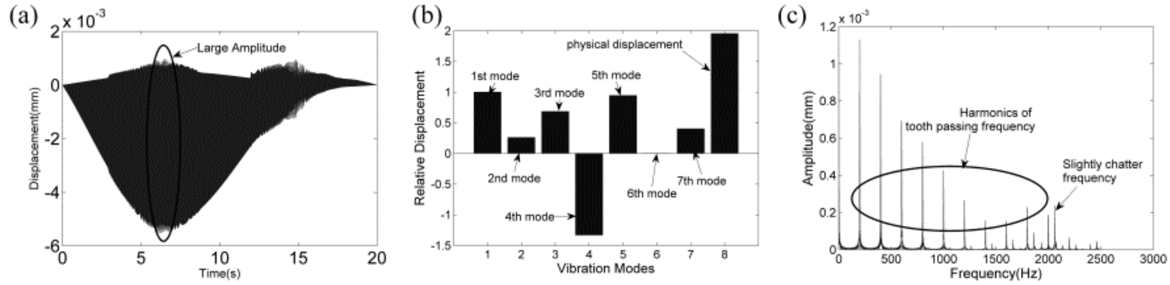

Fig. 2 simulation results (a) structural response (b) the mode contribution at 6s (c) the FFT spectrum

Concluding Remarks

We proposed a dynamic model for milling thin-walled pocket structures. The plate dynamics based on the Kirchhoff hypothesis is adopted to formulate the theoretical model. The difference of this thin wall milling model is studied by numerical simulation. It shows that the contribution of the higher modes may be larger than the first mode, the dynamic features make the chatter may be different from that in the traditional lumped parameters models.

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References

[1] M. Meshreki, H. Attia, J. Kövecses. Development of a new model for the varying dynamics of flexible pocket-structures during machining. *Journal of Manufacturing Science and Engineering* 133, 041002 (2011).

[2] T. Insperger, G. Stépán. Semi‐discretization method for delayed systems. *International Journal for numerical methods in engineering* 55, 503-518 (2002).

[3] X.-H. Long, B. Balachandran. Stability analysis for milling process. *Nonlinear Dynamics* 49, 349-359 (2007).

[4] B. Balachandran, M. Zhao. A mechanics based model for study of dynamics of milling operations. *Meccanica* 35, 89-109 (2000).

[5] S. Ren, X.-H. Long, Y. Qu, G. Meng. A semi-analytical method for stability analysis of milling thin-walled plate. *Meccanica*, accepted.