NONLINEAR IDENTIFICATION OF DAMPING IN LARGE AMPLITUDE VIBRATIONS OF PLATES AND SHELLS

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Summary A nonlinear identification technique is presented to obtain the damping of isotropic and laminated sandwich rectangular plates and curved panels subjected to harmonic excitation as a function of the vibration amplitude. The response of the structures is approximated by (i) reduced-order models with 10 to 100 degrees of freedom and (ii) a single-degree of freedom Duffing oscillator. The method uses experimental frequency-amplitude data and the least-squares technique to identify parameters and reconstruct frequency-response curves by spanning the excitation frequency in the neighbourhood of the lowest natural frequencies. In order to obtain the experimental data, a sophisticated measuring technique has been used. The results reveal a strongly nonlinear correlation between the damping and the vibration amplitude.

Introduction

A challenging concept in nonlinear system identification is the characterization of damping from experimental data. Dissipation is intrinsically a nonlinear phenomenon. The modal damping assumption is a convenient tool that has been extensively used to model dissipation. However, this model generally does not take into account that damping changes with the vibration amplitude. Different nonlinear damping mechanisms have been proposed, the most common being the quadratic damping [1]. Other forms of nonlinear damping include quadratic and cubic powers of relative velocity [2]. Another damping model that is quite often used is the viscoelastic model [3]. Although there have been numerous studies about nonlinear damping, so far none has discussed the change of damping with the vibration amplitude. Therefore, different from previous studies, we propose a nonlinear identification technique to examine the damping behavior of plates and panels during large amplitude vibrations. In order to perform the identification technique, first nonlinear experiments are conducted on isotropic and laminated sandwich plates and curved panels with (i) free edges and (ii) clamped boundary conditions by using a Laser Doppler Vibrometer and a LMS signal processing system to obtain nonlinear experimental frequency-response curves. Then, the frequency-amplitude data obtained from experiments are used as the inputs for the identification scheme and the least squares method is utilized to minimize the error between the measured response and the identified model. It is observed that damping grows very significantly with the vibration amplitude.

Experimental procedure

The non-linear vibration tests have been performed by increasing and decreasing the excitation frequency in very small steps in the frequency neighbourhood of the fundamental mode by using a stepped-sine testing technique [4]. The excitation has been provided by an electro-dynamic shaker, driven by a power amplifier, via a stinger connecting the shaker to the piezoelectric miniature force transducer (B&K type 8203) attached to the structure. The response is then measured by using a very accurate Polytec single point Laser Doppler Vibrometer (sensor head OFV-505 and controller OFV-5000) in order to have non-contact displacement measurement with no introduction of added mass. The time responses have been measured by using a SCADAS III front-end connected to a workstation and the LMS Test.Lab software has been used for signal processing, data analysis and excitation control. In particular, the MIMO Sweep & Stepped Sine Testing application of the LMS system has been utilized to generate the excitation signal and its closed loop control has been used to keep the force constant while the excitation frequency is varied in the neighbourhood of the fundamental frequency.

Modelling and identification method

In order to identify the damping for the experimental data, two procedures are used and their results are compared. First, a very accurate reduced order model with a number of degrees of freedom of the order of 10 to 100 is built and the equations of motions are integrated numerically by using a pseudo-arclength continuation and collocation scheme and the damping is varied until the experimental results are matched. The details on the reduced-order models are given in [4]. In the second procedure, the response of the tested structures is approximated by a single dimensionless nonlinear oscillator with viscous damping, quadratic and cubic non-linearities as follows:

$$r^2\ddot{x} + \zeta\dot{x} + x + \eta_2 x^2 + \eta_3 x^3 = \lambda\cos(t)\ , \tag{1}$$

where x and t are made dimensionless with respect to the structure's thickness and the excitation frequency, respectively. Moreover, ζ is the damping ratio, λ is the dimensionless force, r is the frequency ratio (the ratio between the excitation frequency and the fundamental frequency), and η_2 and η_3 are the dimensionless quadratic and cubic non-linear terms, respectively. Next, the harmonic balance method is applied and the solution of equation (1) is approximated by

$$x \approx x_N = x_0 + \sum_{k=1}^{N}\left[x_{2k-1}\sin kt + x_{2k}\cos kt\right]\ , \tag{2}$$

where N is the chosen order of truncation and x_N is the truncated Fourier series representation of x. A system of algebraic equations is obtained that relates the frequency ratio r to the amplitudes x_N. Next, the identification is conducted by assuming that the vibration amplitude x_N, the frequency ratio r and the harmonic force amplitude are already known for every frequency step from experiments. Therefore, in order to obtain the damping and the non-linear parameters, the following system should be solved for every j-th frequency step, $r^{(j)}$:

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$$\left[2r^{(j)}\mathrm{D}\mathrm{S}_{x^{(j)}} \quad \mathrm{P}_{x^{(j)}} \quad \mathrm{Q}_{x^{(j)}}\right]\cdot\begin{Bmatrix}\zeta\\ \eta_2\\ \eta_3\end{Bmatrix} = \left[-\mathrm{S}_{x^{(j)}} - r^{(j)}\mathrm{D}^2\mathrm{S}_{x^{(j)}} + \mathrm{S}_{f^{(j)}}\right], \quad \mathrm{D}=\begin{pmatrix}0 & 0 & \cdots & 0\\ 0 & \mathrm{D}_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathrm{D}_N\end{pmatrix}, \ \mathrm{D}_k = \begin{pmatrix}0 & -k\\ k & 0\end{pmatrix}, k = 1,\ldots,N \ , \ j = [\![1:m]\!]. \tag{3}$$

where m is the number of excitation frequency steps for which the experimental data was obtained at a specific excitation level. S_x, P_x and Q_x are vectors comprising truncated Fourier coefficients of x, x^2 and x^3, respectively, and S_f is the dimensionless force vector. System (3) is over-constrained, since it contains $(2N+1)\times m$ equations. Therefore, in order to obtain the damping ratio and the non-linear parameters the least squares technique has been applied.

Non-linear identification results

The procedure outlined in the previous section has been applied to (i) AISI 304 stainless steel plate with 0.25×0.24×0.0005m dimensions bolted to a AISI 410 stainless steel rectangular frame exhibiting clamped boundary conditions; (ii) AISI 304 stainless steel circular cylindrical panel (0.199×0.132×0.0003m) inserted in a heavy rectangular steel frame made having V-grooves designed to hold the panel and to avoid displacement of the edges; (iii) a stainless steel rectangular plate (0.3×0.45×0.0008m) with free-edge boundary conditions; (iv) a free-edge sandwich plate (0.46×0.9×0.0033m) with Carbon/Epoxy skins having (0/90) stacking sequence and a DIAB® Divinycell foam core; (v) a second free-edge sandwich plate (0.46×0.9×0.0036m) with Carbon/Epoxy skins having (0/90) lay-up and a PLASCORE® PN2 aramid fiber honeycomb paper core. For the clamped cases the response is measured at the center while for free edges the response is measured at one of the corners. Figures 1(a), 1(b) and 1(c) compare the experimental frequency response curves for different excitation levels and the identified ones for three cases: the free edge rectangular plate, the clamped plate and the curved panel, respectively. It can be observed that the identification (red *) is reasonably accurate and the identified amplitudes are perfectly following the experimental data (blue •) predicting hardening response in case of the flat plates and softening for the curved panel. Figure 1(d) depicts the evolution of damping (normalized with respect to the small-amplitude linear damping) with the normalized peak amplitude for all studied cases (including the two sandwich plates). It is interesting to see that the damping ratio varies nonlinearly with the increase of the peak vibration amplitude. It is evident that for the supported plate and sandwich free-edge plates, the damping could increase more than 250% for vibration amplitudes greater than 1.5 times the thickness. This increase is about 50% for the curved panel and for the same vibration amplitude. However, for the completely free steel plate the increase in damping is around 280% for vibrations greater than 3 times the thickness. This behavior confirms the presence of large dissipation during large-amplitude vibrations.

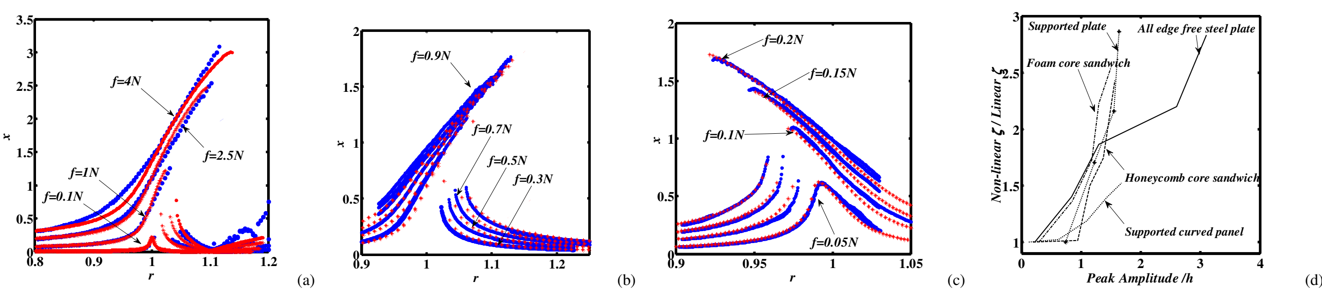

Fig. 1. (a) Comparison between the experimental and identified curves for the tested free-edge isotropic rectangular plate; (b) comparison between the experimental and identified frequency-amplitude curves for the tested clamped rectangular plate; (c) comparison between the experimental and identified curves for the tested clamped curved panel; (d) nonlinear variation of damping (normalized with respect to the linear damping) versus the peak vibration amplitude (normalized with respect to the thickness) for all the studied cases. In (a), (b) and (c) red * denote harmonic balance identification while blue • indicate experimental results.

Conclusions

A nonlinear identification technique is presented to track the evolution of damping during large-amplitude vibrations. The identified damping parameters confirm the presence of a strongly nonlinear correlation between damping and vibration amplitude in plates and curved panels. Particularly, it was found that plates and panels exhibit much larger dissipation during large-amplitude vibrations than in case of small-amplitude vibrations. Specifically, the damping increases with the vibration amplitude well over twice the linear damping ratio for the isotropic and sandwich plates experimentally investigated, and over 50 % for the circular cylindrical panel.

References

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