Delayed Tyre Model in Vehicle Shimmy

Tian Mi*, Gábor Stépán**, Dénes Takács*** and Nan Chen *
*School of Mechanical Engineering, Southeast University, Nanjing, China
**Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary
***MTA-BME Research Group on Dynamics of Machines and Vehicles, Budapest, Hungary

Summary. A 3 DoF vehicle model with dependent suspension is investigated. The two front wheels are equipped with elastic tyres. The lateral forces acting at the contact region between the wheels and the ground are calculated by means of the so-called delayed tyre model. Parameter domains are explored where shimmy may occur in this vehicle model after the loss of stability of straightforward motion.

Problem Definition

Shimmy is a nonlinear vibration which occurs in various cases from landing gears of airplanes, steered wheels of cars and motorcycles to skateboards and supermarket trolleys. Vehicle shimmy is often thought to be the vibration of front wheels around kingpins, which can deteriorate vehicle handling and cause danger.
This phenomenon has attracted much attention since the early 20th century. The vast literature covers topics from the linear models to the advanced nonlinear ones considering elastic tyre, clearance in the steering system, friction, geometric nonlinearities, and so on [1]-[6]. The problem is still far from being fully solved, especially with the appearance of new vehicle constructions, like hub motors, skateboards, dual-wheel scooters, etc.

Mechanical Model

The 3 DoF shimmy model [4] is used, which consists of the shimmy angles $\theta_{1,2}$ of front wheels, and the swing angle φ of the dependent suspension (see Fig. 1 and 2). The vehicle body is assumed to be fixed running straightforward with uniform speed v, that is, there is no steering angle input. All the further geometric and structural parameters including the stiffness values $k_{1,2,3,z}$, damping values $c_{1,2,3,e}$, wheel track l, wheel rolling radius R and the wheel mass m are shown in the figures. The wheel mass moment of inertia is described by the matrix $\mathrm{diag}[\mathrm{J_d},\ \mathrm{J_0},\ \mathrm{J_d}]$, the specific lateral stiffness of the tyre is denoted by k.

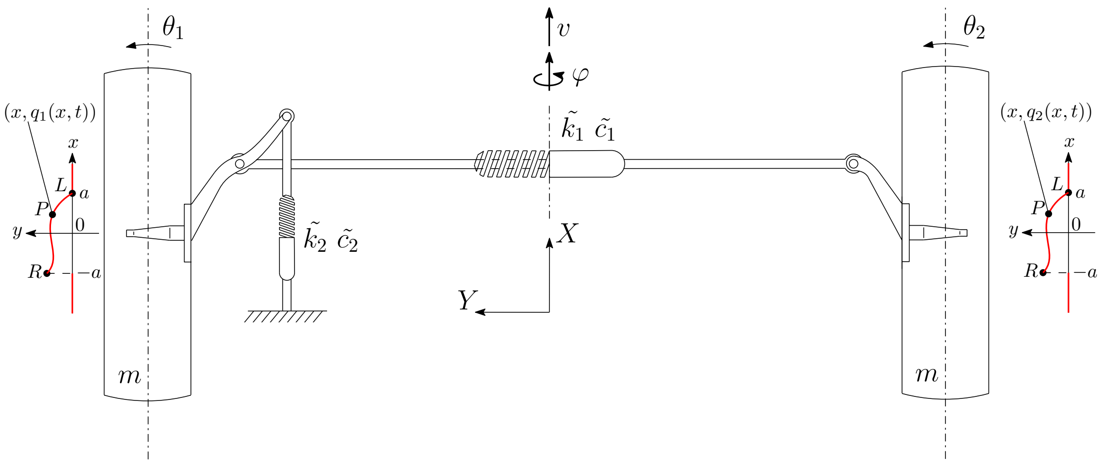

Figure 1: Mechanical model (top view) and brush model of elastic tyre.

The equations of motion assume the form:

$$\begin{cases} (J_d+mr^2)\ddot{\theta}_1(t)-(J_d+m\frac{l}{2}r)\gamma\ddot{\varphi}(t)+(c_1+c_2+c_e)\dot{\theta}_1(t)+(k_1+k_2)\theta_1-c_1\dot{\theta}_2(t)-k_1\theta_2(t)+J_0\frac{v}{R}\dot{\varphi}_1(t) \\ -k_z\frac{l}{2}r\,\gamma\varphi(t)=k\int_{-a}^{a}(x-R\gamma)q_1(x,t)dx, \\ (J_d+mr^2)\ddot{\theta}_2(t)-(J_d+m\frac{l}{2}r)\gamma\ddot{\varphi}(t)-c_1\dot{\theta}_1(t)-k_1\theta_1+(c_1+c_e)\dot{\theta}_2(t)+k_1\theta_2(t)+J_0\frac{v}{R}\dot{\varphi}(t) \\ -k_z\frac{l}{2}r\,\gamma\varphi(t)=k\int_{-a}^{a}(x-R\gamma)q_2(x,t)dx, \\ 2\Big(J_d+m(\frac{l}{2})^2\Big)\ddot{\varphi}(t)-(J_d+m\frac{l}{2}r)\gamma\Big(\ddot{\theta}_1(t)+\ddot{\theta}_2(t)\Big)-J_0\frac{v}{R}\Big(\dot{\theta}_1(t)+\dot{\theta}_2(t)\Big)-k_z\frac{l}{2}r\,\gamma\Big(\theta_1(t)+\theta_2(t)\Big) \\ +c_3\dot{\varphi}(t)+\Big(k_3+2k_z(\frac{l}{2})^2\Big)\varphi(t)=k\Big(\int_{-a}^{a}q_1(x,t)dx+\int_{-a}^{a}q_2(x,t)dx\Big)R, \end{cases} \tag{1}$$

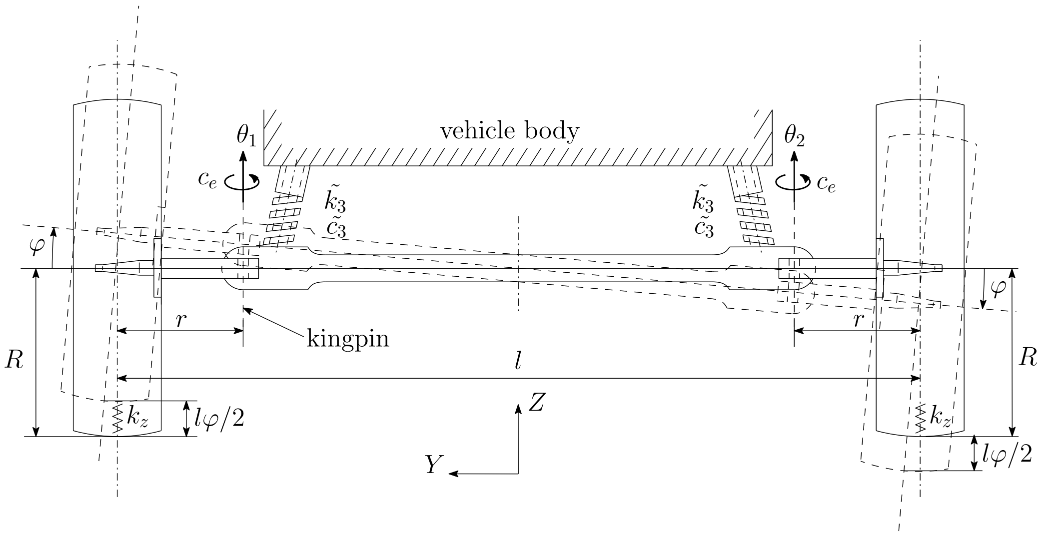

Figure 2: Mechanical model (rear view).

where $q_{1,2}(x,t)$ stand for the lateral deformations of the tyre within the contact region $x \in [-a,\, a]$, with a being the half contact length (see Fig. 1). The position of a tyre contact point P in the ground can be expressed in linearized form as

$$\begin{cases} X(x,t) = vt - r\,\theta_{1,2} + x, \\ Y(x,t) = \dfrac{l}{2} + q_{1,2}(x,t) + (x - R\gamma)\theta_{1,2} + R\varphi. \end{cases} \tag{2}$$

The contact points have zero velocities. This nonholonomic constraint leads to partial differential equations (PDE) with respect to $q_{1,2}(x,t)$, which have travelling wave solutions according to

$$\begin{pmatrix} X(x,t) \\ Y(x,t) \end{pmatrix} = \begin{pmatrix} X(a,t-\tau) \\ Y(a,t-\tau) \end{pmatrix}, \tag{3}$$

where $\tau \approx (a-x)/v$ is the time needed from the leading contact point L to reach an actual position at P. In case of a brush model [1] with boundary condition $q_{1,2}(a,t) = 0$, equations (2) and (3) lead to

$$q_{1,2}(x,t) = v\tau\theta_{1,2}(t) - (a - R\gamma)\Big(\theta_{1,2}(t) - \theta_{1,2}(t-\tau)\Big) - R\Big(\varphi(t) - \varphi(t-\tau)\Big). \tag{4}$$

Its substitution into the equations of motion (1) results a system of delay differential equations (DDE).

Stability Analysis

The trivial solution of the DDE corresponds to the straightforward motion of the vehicle with wheels rolling straight without vibration. The stability of this motion can be studied with the methods presented in [5] and [6]. In order to study the related transcendent characteristic function, the pseudospectral tau approximation [7] is used to produce the stability charts in the plane of relevant parameters like speed v, wheel track l and caster angle γ.

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