Small-scale counter-rotating Darrieus wind turbine

Shalimova Ekaterina*, Klimina Liubov*, Samsonov Vitaly*, and Lin Ching-Huei**
*Institute of Mechanics, Lomonosov Moscow State University, Moscow, Russia
**Electrical Engineering Department, Chien Hsin University of Science and Technology, Taoyuan City, Taiwan

Summary. The mathematical model of a counter-rotating Darrieus vertical axis wind turbine is constructed. Quasi-steady model of aerodynamic action is used. It is supposed that an electrical generator is connected into a local circuit with changeable external resistance. Equations of model are simplified via averaging of the aerodynamic torque. Parametrical analysis of operation modes corresponding to stable fixed points of the simplified model equations is performed.

Introduction

Motion of a small-scale Darrieus counter-rotating vertical axis wind turbine (VAWT) in a steady wind flow is studied. A closed few-parametrical mathematical model that takes into account changeable electrical load in the local circuit of a generator is constructed. The similar model for a single-turbine Darrieus setup was discussed in [1]. Such kind of model allows performing detailed parametrical analysis of operation modes of the turbine. Moreover, the model is modified by averaging of the aerodynamic torque per revolution. This simplification allows to construct a bifurcation diagram of operation angular speeds depending on an external load coefficient.

Description of the system

Dynamics of a small-scale counter-rotating Darrieus can be described by the following dimensionless equations (a derivative with respect to a dimensionless time is denoted by a dot):

$$\begin{cases} \ddot{\vartheta} = \varepsilon\left(f_{aero}(\vartheta,\dot{\vartheta}) - c(\dot{\vartheta}+\dot{\varphi})\right); \\ \ddot{\varphi} = \varepsilon\left(f_{aero}(\varphi,\dot{\varphi}) - c(\dot{\vartheta}+\dot{\varphi})\right). \end{cases} \quad (1)$$

$$f_{aero}(x,y) = \sum_{i=1}^{3}\sqrt{d_i^2 + l_i^2}\left(C_l(\alpha_i)l_i - C_d(\alpha_i)d_i\right); \quad l_i = \cos\left(x + \frac{2\pi}{3}i\right), \quad d_i = \left(y + \sin\left(x + \frac{2\pi}{3}i\right)\right); \quad \alpha_i = \mathrm{arctg}\left(\frac{l_i}{d_i}\right);$$

$$c = \frac{2C}{V\rho Sr^2(\sigma + R)} > 0; \quad \varepsilon = \frac{\rho Sr^3}{2J} > 0.$$

Here ϑ and φ are angles of orientation of two turbines of the device with respect to the direction of the wind flow. These two angles are counted in the opposite directions (fig. 1*a*). Each turbine has three similar blades. $\varepsilon = 0.5\rho Sr^3 J^{-1}$ is supposed to be a small parameter (ρ is the air density, r is the radius of each turbine, S is the characteristic area of wings, J is the moment of inertia of each turbine around the axis of rotation). α_i is an instantaneous angle of attack for a blade, $C_d(\alpha)$, $C_l(\alpha)$ are drag and lift aerodynamic coefficients.

It is supposed that the rotor of generator is joined to the shaft of one turbine and the stator is joined to the shaft of the second one. Thus, the coefficient c is responsible for an electrical load upon the generator (V is the wind speed, C is the electromechanical coupling coefficient, σ is the inner resistance of the generator, R is the external resistance in the circuit of the generator).

Suppose that ε is sufficiently small and the angular speeds of both turbines are sufficiently large, so that fluctuations of the angular speed during one revolution can be neglected. Due to such assumption substitute the aerodynamic torque in (1) by an averaged torque per a revolution:

$$\begin{cases} \dot{\omega}_1 = \varepsilon\left(\Psi(\omega_1) - c(\omega_1+\omega_2)\right); \\ \dot{\omega}_2 = \varepsilon\left(\Psi(\omega_2) - c(\omega_1+\omega_2)\right). \end{cases} \quad \Psi(\omega) = \frac{1}{2\pi}\int_0^{2\pi} f_{aero}(x,\omega)dx. \quad (2)$$

A qualitative view of $\Psi(\Omega)$ for a wide range of turbines is that one shown in the fig. 1*b*.

An operation mode of the device corresponds to a stable fixed point $\{\Omega_1, \Omega_2\}$ of the system (2).

Operation modes

Fixed points $\{\Omega_1, \Omega_2\}$ of (2) can be found from the following equations:

$$\Psi(\Omega_1) = \Psi(\Omega_2); \quad c = \frac{\Psi(\Omega_1)}{\Omega_1 + \Omega_2}. \quad (3)$$

Due to the symmetry of (2), it is enough to discuss the case $\Omega_1 \le \Omega_2$. A qualitative bifurcation diagram of $\Omega_1(c)$, $\Omega_2(c)$ is represented in the fig. 1*c*.

One branch of solutions of (3) is $\Omega_1(c) = \Omega_2(c) = \Omega(c)$ (the black curve in the fig. 1*c*). Moreover, there are three sets of branches for which $\Omega_1 \neq \Omega_2$, but $\Psi(\Omega_1) = \Psi(\Omega_2)$. These branches are qualitatively shown by colored curves in the fig. 1*c* (each branch $\{\Omega_1(c), \Omega_2(c)\}$ is marked by an individual color).

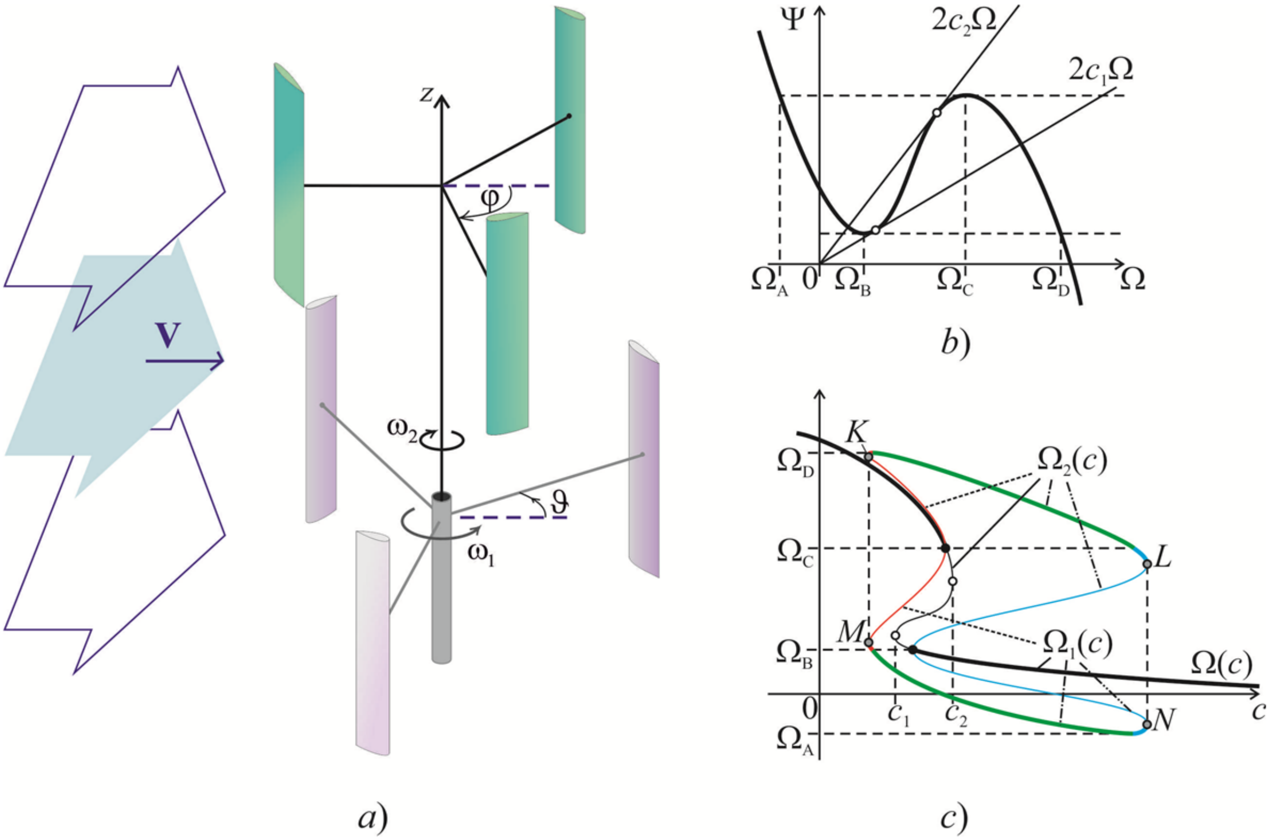

Fig. 1 *a*) The scheme of the double Darrieus wind turbine; *b*) The qualitative view of $\Psi(\Omega)$; *c*) The qualitative bifurcation diagram.

The following conditions are sufficient for asymptotical stability of a fixed point of (2):

$$\begin{cases} 2c > \Psi_1' + \Psi_2'; \\ c\left(\Psi_1' + \Psi_2'\right) < \Psi_1'\Psi_2', \end{cases} \quad \text{where} \quad \Psi_i' = \left.\frac{d\Psi}{d\omega}\right|_{\omega=\Omega_i} \tag{4}$$

If an unequality that is opposite to any of unequlites (4) is fulfilled, then the corresponding fixed point is unstable. The conditions (4) are fulfilled only for branches marked with bold lines in the fig. 1*c*. Thus, the sub-branches $\{\Omega(c) > \Omega_C\}$ and $\{\Omega(c) > \Omega_C\}$ of the branch $\Omega_1 = \Omega_2 = \Omega(c)$ correspond to stable fixed points; and also the sub-branches for which $\{\Omega_1(c) \in MN, \Omega_2(c) \in KL\}$; the other sub-branches correspond to unstable fixed points (for the points *K*, *L*, *M*, *N* the linearized system is not enough to check stability).

Discussion

Comparison with a single-turbine VAWT [1] yields the following qualitatively different features of behavior:

1) Unstable part of the branch $\Omega(c)$ is wider for a counter-rotating VAWT than for a classical one.

2) For a counter-rotating VAWT, additional types of operation modes are present. Moreover, for the range $c > c_2$ of external load coefficient, the trapped power at these additional modes can be significantly higher than that for the mode with equal angular speeds of two turbines.

This work was partially supported by the Russian Foundation for Basic Research, projects NN 15-01-06970, 16-31-00374, and 17-08-01366.

References

[1] Klimina L., Lokshin B., Samsonov V. (2009). Parametrical analysis of the behaviour of an aerodynamic pendulum with vertical axis of rotation // Modelling, Simulation and Control of Nonlinear Engineering Dynamical Systems. State-of-the-Art, Perspectives and Applications. Springer. pp. 211–220.