Dynamic analysis of a flexible manipulator with embedded PZT actuators based on FE method

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Summary. This paper builds a Finite Element (FE) model of a flexible manipulator with embedded PZT actuators. Propose a Corrected Rebuild Reduced Model (CRRM) to make the dynamic characteristics of the model more consistent with reality and facilitate control design. The CRRM considers the holding torque of the driving motor of the manipulator, and also eliminates the response divergence induced by the fault of the mass matrix of the FE model. In order to reduce the dimensions and maintain the precision of the FE model, an Iterated Improved Reduction System (IIRS) method is adopted. Furthermore, a normal LQR controller is designed based on the output function of the improved model.

Introduction

Lightweight and large scale flexible structures are utilized in many fields. Some of them exhibit excellent performance in dynamics when integrated with intelligent components. Especially in space application, manipulators with flexible links and joints are widely used. These structures have the advantages of lower energy consumption in operation and higher speed in motivation. However, they always have small damping and low natural frequencies[1]. These characteristics may induce residual vibration with large amplitude. Additionally, the vibration and rigid motion of the manipulators are highly coupled to each other[2]. It is a challenge to control the vibration and maintain the rigid motion of the system at the same time.

This paper presents an approach to suppress the vibration of a manipulator with both a flexible link and joint based on the FE method. A pair of embedded PZT actuators are considered in the FE model. The IIRS method is introduced to reduce the order of the FE model. The high precision characteristic of the method helps the reduced model to maintain consistent performance in the effective frequency band compared to the original. Furthermore, in order to fit the dynamic characteristics of the model to reality, excess inertia instead of holding torque is added to the rotor of the drive motor. A fault of the mass matrix of the FE model induces the response of the FE model to be divergent. A reconstructed method is presented and deals with this problem effectively. The function of the reconstructed model is transformed into the expression form with output variables. Then, it is easy to design an optimal controller based on the output function. Simulation results show good performance in the vibration suppression of the manipulators.

Dynamic modelling

A manipulator is primarily composed of an electric motor, elastic joint, follow-up unit, PZT actuators, mounting base for the actuators, flexible link and a tip mass. The manipulator is driven by the electric motor, and the follow-up unit is connected to the rotor of the motor by the elastic joints. The link is a flexible beam. A pair of actuators are embedded in the mounting base which is fixed to the root segment of the beam. In some cases, the mounting base can be considered as a part of the flexible beam. At the end of the link, a tip mass is considered. This manipulator demonstrates a general case under the conditions of having one link. The outline of the constituent parts of the manipulator can be designed arbitrarily. Therefore, the model can be used to describe a vast majority of the structures with slender rotating beams. The motion of the manipulator is planar. The typical Euler-Bornoulli assumption is made for the flexible beam.

The kinetic energy and potential energy of the entire system is generated by the rotator of the electrical motor, flexible joint, follow-up unit, flexible beam and tip mass. According to the Lagrange and FE method, the entire model of the system is described as

$$\begin{bmatrix} J_r & 0 & \mathbf{0} \\ 0 & \bar{m}_{\theta\theta} & \bar{\mathbf{M}}_{\theta q} \\ \mathbf{0} & \left(\bar{\mathbf{M}}_{\theta q}\right)^T & \bar{\mathbf{M}}_{qq} \end{bmatrix} \begin{bmatrix} \ddot{\beta} \\ \ddot{\theta} \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} k_J & -k_J & \mathbf{0} \\ -k_J & k_J & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{K}}_{qq} \end{bmatrix} \begin{bmatrix} \beta \\ \theta \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} M_r \\ 0 \\ \bar{\mathbf{Q}}_q \end{bmatrix} \tag{1}$$

where β, θ are the rotation angles of the rotor and the mounting base respectively, J_r, k_J $\bar{m}_{\theta\theta}$ are system parameters, $\bar{\mathbf{M}}_{qq}$, $\bar{\mathbf{K}}_{qq}$ are discrete mass and stiff matrixes of the flexible beam with embedded PZT actuators and tip mass, $\bar{\mathbf{M}}_{\theta q}$ is the coupling term of mass matrix, M_r, $\bar{\mathbf{Q}}_q$ are general force of the system.

Model reduction

The FE models of the system are always high dimensions. In order to improve the computational efficiency, it is necessary to reduce the dimensions of the system. The Iterated IRS method [3] has good performance and is easy to work with. Rewriting the Eq. (1) to partitioned form obtains

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_m \\ \ddot{\mathbf{x}}_s \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{x}_m \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} \mathbf{f}_m \\ \mathbf{0} \end{bmatrix} \tag{2}$$

where the subscripts m and s relate to the master and slave coordinates respectively. The reduced model can be described as

$$\mathbf{M}_r \ddot{\mathbf{x}}_m + \mathbf{K}_r \mathbf{x}_m = \mathbf{f}_r \tag{3}$$

where $\mathbf{M}_r$ and $\mathbf{K}_r$ are the mass and stiff matrixes after *i*th iteration.

Control design and dynamic response

Vibration control focuses on suppressing the deformation induced by fast variables, and the slow variables just as the rotation angle is β and θ needn't be considered in control function. Combining the generalized inverse matrix and transferring the controlled function into discrete the output function

$$\mathbf{y}(k+1) = \bar{\mathbf{A}}_d y(k) + \bar{\mathbf{B}}_d \mathbf{u}(k) + \mathbf{H}_d \mathbf{w}(k) \tag{4}$$

The form of Eq.(4) is a standard state function, it facilitates designing a state feedback controller with the LQR optimal method. One obtains

$$\mathbf{u}(k) = -\mathbf{L}\mathbf{y}(k) \tag{5}$$

where $\mathbf{L}$ is the matrix of the feedback control coefficient, which can be calculated by Matlab function [L,S,e]=dlqr($\bar{\mathbf{A}}_d$, $\bar{\mathbf{B}}_d$, Q, R), and Q and R are weighted matrixes of $\mathbf{y}$ and $\mathbf{u}$ respectively.

By substituting system parameters into functions, and valuating the weighted matrixes as $\mathbf{Q} = 10^3 \cdot \mathbf{I}_{2(N^r-2)}$, $\mathbf{R} = \mathbf{I}_3$, the controller coefficient L^* can be calculated by the LQR optimal equation solver. The responses of the controlled system are listed in Fig. 1-Fig. 4.

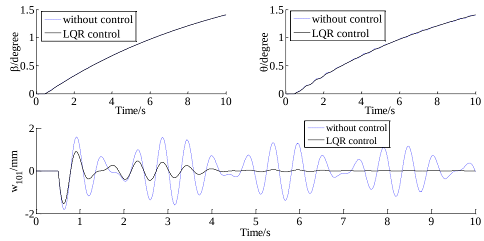

Fig. 1 Response of the controlled system with impulse excitation

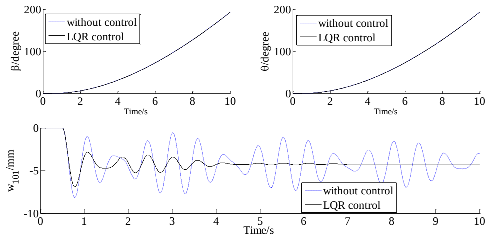

Fig. 2 Response of the controlled system with step excitation

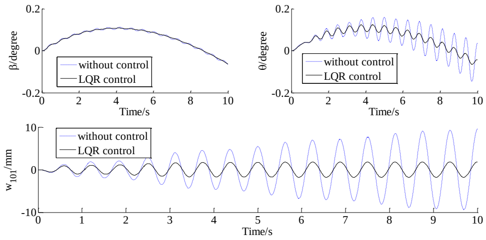

Fig. 3 Response of the controlled system with harmonic excitation

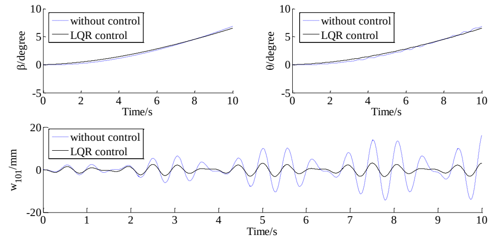

Fig. 4 Response of the controlled system with hybrid excitation

Conclusions

This paper solves the vibration problem for a manipulator with a flexible link and joint. The research route of the paper is summed up as follows: (1) Establishing an FE model of the manipulator with a flexible link and joint. (2) Reducing the dimensions of the FE model with a high precision model reduction method. (3) Proposing the CRRM to update the reduced FE model, and fitting the dynamic characteristics of the system to reality. (4) Presenting a model reconstruction method to correct the fault of the CRRM mass matrix, and ensuring the response of the system is not divergent. (5) Establishing the state function of the output system, designing an LQR optimal controller for the reconstructed model, and performing simulations.

All simulation results indicate good performance in the aspects of dynamic characteristics and controlled effect. The response of the reconstructed model without control shows that the joint of the manipulator can prevent high frequency vibration being transmitting to the flexible link. The vibration is almost induced by the first two order modes of the reconstructed model. The vibration phenomenon is in accord with reality. The controller design only refers to the signal of the vibration, not the rigid motivation of the manipulator. Therefore, the response of the controlled system has good performance in vibration suppression and maintains excellent performance in rigid motivation. The method of analysis in this paper can be used to guide practical application.

References

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