Torsional vibration damper design using Augmented Lagrangian Particle Swarm Optimization

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Summary. Torsional vibration dampers are comfort relevant components of automotive powertrains. Transferring the requirements imposed on their design into mathematical models leads to a nonlinear constrained optimization task. Given this problem's structure, stochastic methods like the Augmented Lagrangian Particle Swarm Optimization (ALPSO) algorithm [1] offer suitable tools to find solutions. This procedure showed desirable convergence properties on various test functions and was used successfully to solve real-world problems in engineering [1, 2]. To handle the comparably high problem dimension and the large amount of constraints, a two-stage optimization process has been implemented. The first pre-conditioning step improves the initial population's quality by applying gradient-based search methods to a reduced problem. ALPSO is then used in the subsequent main optimization of the full problem.

Introduction

In recent years, growing computational power has raised increasing interest in using numerical optimization methods to automate engineering design processes. Solving nonlinear differential equations is often required to evaluate a draft's performance. In this case, the target function f and the equality and in equality constraints $\boldsymbol{g}$ and $\boldsymbol{h}$ might not only depend on the parameter vector $\boldsymbol{x}$, but also on the time t, the state variables $\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{x}, t)$, their respective time derivatives $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ and a given external excitation $\boldsymbol{e} = \boldsymbol{e}(t)$. The optimization problem is stated as

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{e}, t), \qquad \boldsymbol{x} \in D \subseteq \mathbb{R}^n \text{ and } f: \mathbb{R}^n \to \mathbb{R} \tag{1}$$

$$\text{subject to } \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{e}, t) = \boldsymbol{0}, \qquad \boldsymbol{g}: \mathbb{R}^n \to \mathbb{R}^{m_e} \tag{2}$$

$$\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{e}, t) \leq \boldsymbol{0}, \qquad \boldsymbol{h}: \mathbb{R}^n \to \mathbb{R}^{m_i}. \tag{3}$$

Gradient-based search methods like Sequential Quadratic Programming (SQP) are powerful tools to solve constrained problems, but they require continuous and differentiable target and constraint functions. Gradient-free methods like ALPSO, which adds the Augmented Lagrangian Multiplier Method (ALM) [3] to the basic PSO procedure [4], avoid these restrictions. This results in a very robust algorithm, capable of finding global optima even if the problem is ill-conditioned. It showed competitive or superior convergence properties to other stochastic methods [1] and has also been applied successfully to several real-world problems in engineering [1, 2].

ALPSO algorithm outline

Constraints are often incorporated into the objective function by adding a penalty function multiplied with a penalty factor. If the constraints are violated, the target function value is increased. The problem is transformed into an unconstrained optimization task. However, the solution of the new problem will only converge to a solution of the original problem if the penalty factors become infinite. ALM circumvents ill-conditioning by applying the penalty method to the problem's Lagrangrian instead of the target function. This leads to the augmented Lagrangian

$$\mathcal{L}_A = f + \sum_{i=1}^{m_e+m_i} \lambda_i \, \sigma_i + \sum_{i=1}^{m_e+m_i} r_{p,i} \, \sigma_i^2, \quad \sigma_i = \begin{cases} g_i(\boldsymbol{x}), \ i = 1..m_e \\ \max\left(h_i(\boldsymbol{x}), -\frac{\lambda_i}{2r_{p,i}}\right), \ i = m_e + 1..m_e + m_i \end{cases} \tag{4}$$

$\boldsymbol{\lambda}$ denotes the vector of Lagrangian multipliers and $\boldsymbol{r}_p > \boldsymbol{0}$ are the penalty factors. $\boldsymbol{\sigma}$ enables the incorporation of inequality constraints into the method [5]. The ALPSO algorithm uses the basic PSO procedure [4]

$$\boldsymbol{x}_j^{k+1} = \boldsymbol{x}_j^k + \boldsymbol{\Delta x}_j^{k+1} \tag{6}$$

$$\boldsymbol{\Delta x}_j^{k+1} = w\boldsymbol{\Delta x}_j^k + d_1 r_{1,j}^{k+1}\left(\boldsymbol{x}_j^{best} - \boldsymbol{x}_j^k\right) + d_2 r_{2,j}^{k+1}\left(\boldsymbol{x}_{swarm}^{best} - \boldsymbol{x}_j^k\right) \tag{7}$$

to find minimizers to the augmented Lagrangian. PSO simulates a bird flock moving through the design space on a search for the optimal position. $\boldsymbol{x}_j^k$ denotes the location of particle j in iteration k. The particle's velocity $\boldsymbol{\Delta x}_j^{k+1}$ defines its translation from iteration k to $k+1$. It is calculated using the velocity in the previous step $\boldsymbol{\Delta x}_j^k$, the distance to the particle's best position in the past $\boldsymbol{x}_j^{best}$ and the total best position in the swarm $\boldsymbol{x}_{swarm}^{best}$. w, d_1 and d_2 are parameters controlling the swarms behaviour. $r_{1,j}^{k+1}$ and $r_{2,j}^{k+1}$ are random numbers uniformly distributed in $[0,1]$ representing the method's stochastic nature.

Torsional vibration damper design problem

Problem statement

Torsional vibration dampers reduce the combustion engine's rotational irregularities before they are transmitted to the powertrain. This is conventionally achieved by splitting the flywheel into two and connecting the two sides with soft springs. Centrifugal pendulum-type absorbers can be applied in addition for further decreasing of vibrations. The design problem consists of defining a set of springs and pendulums which is mountable, fits into a given assembly space, withstands operation loads and provides sufficient isolation. These requirements are supposed to be fulfilled by a system with preferably low mass moment of inertia Θ which leads to the associated optimization problem

$$\min_{\boldsymbol{x}} \Theta(\boldsymbol{x}) \tag{8}$$
$$\text{subject to } \boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u} \tag{9}$$
$$\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b} \tag{10}$$
$$\boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{0} \tag{11}$$
$$\boldsymbol{k}(\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{e}, t) \leq \boldsymbol{0}. \tag{12}$$

Here $\boldsymbol{l}$ and $\boldsymbol{u}$ are simple bounds and define basic limitations to the design space D. The linear constraints (10) represent requirements on the assembly. The nonlinear constraints are divided into two classes. The first group (11) is only depending on the parameter vector. It contains restrictions on durability and features like spring characteristics and pendulum geometries and paths. The second group (12) ensures the damper's actual function and is based on dynamic simulations. The calculation results for different typical driving situations like full-load acceleration, engine start or idling are used to determine whether the powertrains vibrational behaviour is tolerable or not. Note that there are no equality constraints to this specific problem.

Optimization method

The problem dimension is comparably high, given that $\boldsymbol{x} \in \mathbb{R}^{20..35}$ depending on the damper concept under consideration. The total number of constraints might add up to 100. The feasible region is very small and it is difficult to achieve a good design space coverage with an acceptable amount of particles. For these reasons, a two-stage optimization process has been implemented. In a first step, a set of particles is created and distributed uniformly to random locations in the design space D. These positions are then used as starting points for the optimization of a reduced problem with a SQP algorithm. The simulation-based constraints $\boldsymbol{k}$ are neglected in this step and the target function is set to be constant. The solutions to this reduced problem are then used as new initial locations for the swarm and the full optimization problem (8)-(12) is solved using the ALPSO algorithm. The process is depicted in **figure 1**.

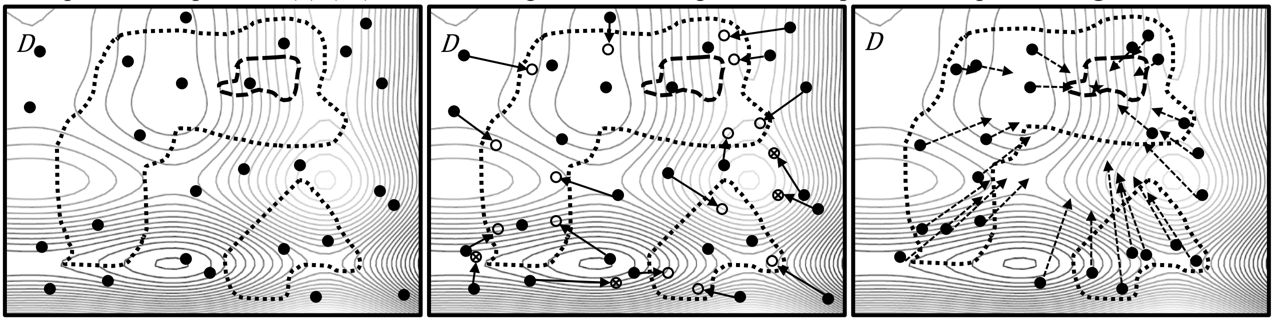

Figure 1: Torsional vibration damper design process: Random initial population (left), pre-conditioning using SQP (middle) and main optimization with ALPSO (right). The feasible regions are indicated by dotted lines for the reduced problem and dashed lines for the full problem respectively

Conclusions

A two stage optimization process for torsional vibration damper design was implemented. The pre-conditioning step improves feasible region coverage and allows for smaller swarms in the main optimization. This reduces computation time without diminishing result quality. In cases where no feasible solution is known in advance, the first step might be crucial to be able to identify solutions which fulfil all constraints at all. Even if the solution space is not as heavily restricted, the method will raise convergence rates.

References

[1] Sedlaczek K., Eberhard P. (2006) Using augmented Lagrangian particle swarm optimization for constrained problems in engineering. *Struct Multidisc Optim* **32(4)**:277-286.

[2] Sauermann R. et al. (2010) Particle Swarm Optimization for Automotive Model-Based Calibration. *IFAC Proceedings Volumes* **43(7)**:733-738.

[3] Rockafellar R.T. (1971) New applications of duality in convex programming. *Proc. 4th Conference on Probability*:73-81.

[4] Kennedy J., Eberhart R. (1995) Particle swarm optimization. *Proc. IEEE International Conference on Neural Networks* **4**:1942-1948

[5] Rockafellar R.T. (1974) Augmented Lagrange Multiplier Functions and Duality in Nonconvex Programming. *SIAM Journal on Control* **12(2)**: 268-285