Continuation of periodic orbits in symmetric conservative systems: an application to the planar 2k+1 body problem

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Summary. We briefly review the numerical aspects of a continuation procedure for conservative and symmetric systems [1, 2]. As an application we continue a classical horseshoe shaped orbit in the 3 body problem as some of the masses are varied to investigate the stability and bifurcation behavior of the family. This periodic solution can be generalized to the case of 2k+1 bodies in the plane and its relation to the Maxwell configuration in which a massive body is surrounded by a rotation regular polygon of smaller bodies. In particular, we present a systematic way to produce new families in which the number of bodies is increased by two within an appropriate continuation scheme in a boundary value problem where the symmetric configurations play a prominent role. From the numerical point of view it is a challenging and demanding problem.

Introduction

In conservative and symmetric systems the periodic orbits usually belong to multi-parameter families that make the continuation procedure non-standard (cylinder theorem). The traditional way out is to make use of the associated conserved quantities or symmetries to **reduce** the dimension of the problem. An alternative approach was proposed in [1] in which the dimension of the problem is **enlarged** by adding as many appropriate *unfolding* terms as conserved quantities. In [2] the symmetries and reversibilities present in the model are incorporated in the continuation procedure to study highly symmetric solutions as, for instance, the choreographies in the 3-body problem [3].

Application to the 3 body horseshoe solution

Applying this technique to the well known three body horseshoe orbits of the 3BP we study the stability and bifurcations of a toy model of Janus and Epimetheus dancing solution around Saturn [4].
The initial conditions of the orbits were determined by means of solving a boundary value problem with one free parameter. The numerical solution of the boundary value problem was obtained using the software AUTO. For the numerical analysis we have used the value of 3.5×10^{-4} as mass ratio of some satellite and the planet. In the computed solutions the satellites are in mean motion resonance 1:1 and they librate around a relative equilibria, that is a solution where the distances between the bodies remain constant for all time.

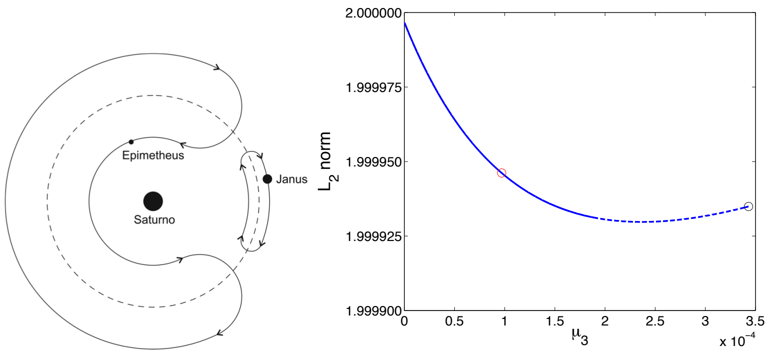

Figure 1: Left: Janus and Epimetheus schematic horseshoe solution around Saturn in a rotating frame. Right: bifurcation diagram as one of the masses is varied with a change of stability along the branch.

Generalization to the 2k+1 body problem

Recently [5] we have extended this horseshoe shaped solution the more symmetric case in which 5 bodies are involved. The system under study corresponds to one conformed by a planet and four satellites of equal mass. We determine a 1 parameter family of time-reversible invariant tori, related with the reversing symmetries of the equations of motion. An appropriate boundary value problem with highly symmetric endpoints has to be continued with adequate resonance conditions. The necessary and sufficient conditions for periodicity of the orbits are discussed.
We show how this family can be connected to the Maxwell configuration in which a massive body is surrounded by a rotation regular polygon of smaller bodies. In particular, we present a systematic way to produce new families in which the number of bodies is increased by two.
From the numerical point of view it is a challenging and demanding problem.

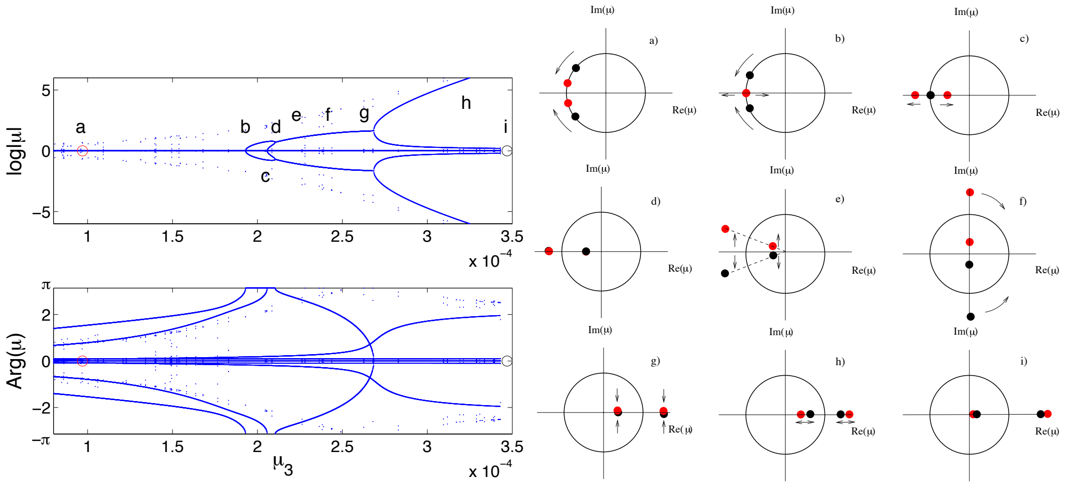

Figure 2: Behavior of the Floquet multipliers along the previous branch in which different hamiltonian bifurcations are found. Left: Numerical results. Right: schematical representation.

Finally, we will discuss the implication of these solutions in a temptative counterexample to Saari's conjecture.

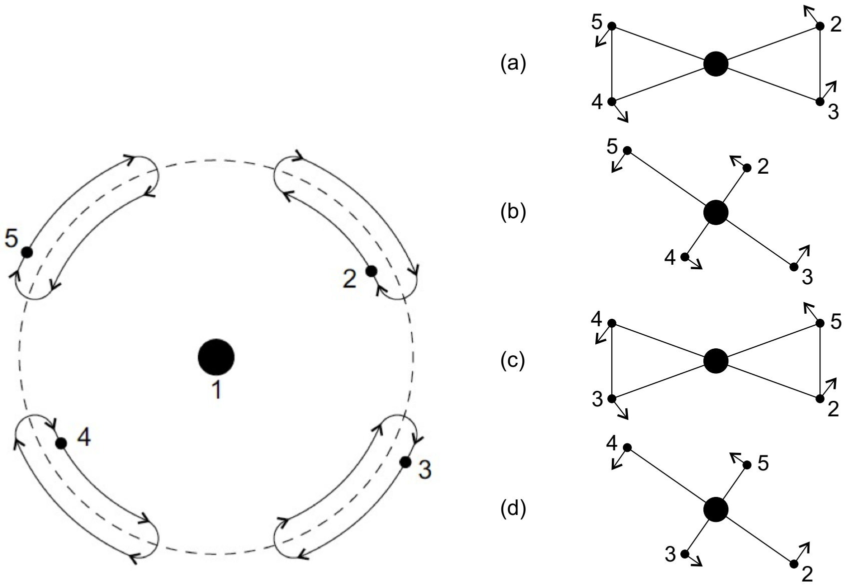

Figure 3: Svhematic representation of the 5 body horsehoe solution ina rotating frame (left) and the symmetric configurations that appear in the BVP formulation of the solution (right).

References

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