Dynamics of the basketball rolling along the rim

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Summary. The dynamics of a basketball is investigated which is rolling on the basketball rim. By assuming rigid bodies, the motion is described by a set of four first-order differential equations. The stationary solutions are determined, which describe the stationary rolling of the ball along the circumference of the rim. For the non-stationary solutions, numerical simulations are performed to determine whether the ball falls from the rim inside or outside the basket.

Introduction

It can be seen often during basketball games that the ball starts rolling around and around the rim, and it is quite unpredictable whether the ball falls down inside or outside the basket. Dynamics of the ball has been investigated from different approaches (see [1], [2]), and the system has many similarities with the motion of golf ball [3]. In this work, a simple model is presented by assuming rigid bodies and rolling contact, which is capable to find and analyse the special motions related to the rolling of the ball around the circumference of the rim.

Mechanical model

The basketball is modelled by a rigid sphere and the rim is modelled by a rigid torus (see Figure 1). The ball has a radius R, a mass m and a mass moment of inertia jmR^2, where $j = 2/3$ is assumed as the mass of the ball is distributed on its surface. The basketball rim has radii R and r and its thickness is a. The gravitational acceleration is denoted by g.

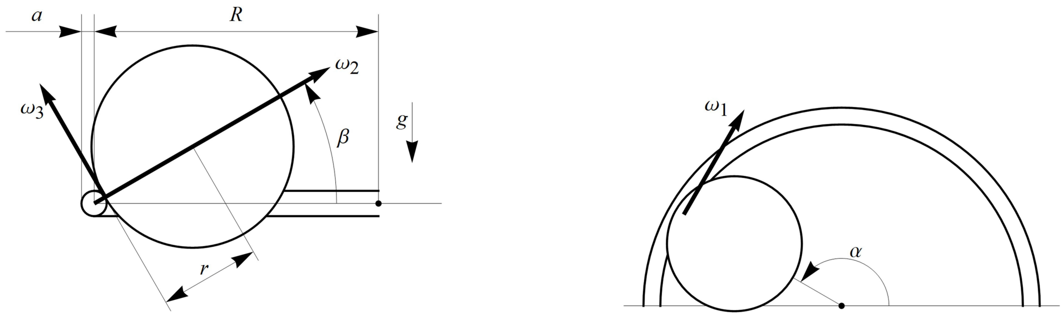

Figure 1: Sketch of the mechanical model.

Equations of motion

The components of the angular velocity of the ball are considered in a coordinate system fixed to the tangent plane of the ball at the contact point (see Figure 1). The *transverse* angular velocity is denoted by ω_1, which component shows the rotation of the ball around the axis tangent to the rim. The *spinning* angular velocity is denoted by ω_2. The *circular* angular velocity is denoted by ω_3 which expresses the rotation of the ball related to the motion around the circumference of the rim. The tilt angle of the ball around the rim is denoted by β, where the value $\beta = \pi/2$ corresponds to the position of the ball on the top of the rim. By assuming that the ball is rolling, it can be derived that the equations of motion of system are

$$
\begin{aligned}
\dot{\omega}_1 &= \frac{(1+j)r\omega_3^2 \sin\beta - jr\omega_2\omega_3\cos\beta}{(1+j)(R-(a+r)\cos\beta)} - \frac{g\cos\beta}{r(1+j)},\\
\dot{\omega}_2 &= \frac{rR\omega_1\omega_3}{(a+r)(R-(a+r)\cos\beta)},\\
\dot{\omega}_3 &= -\frac{r\omega_1\omega_3\sin\beta}{R-(a+r)\cos\beta} - \frac{jr\omega_1\omega_2}{(a+r)(1+j)},\\
\dot{\beta} &= \frac{r}{a+r}\omega_1.
\end{aligned}
\tag{1}
$$

Due to the symmetries of the geometry, the orientation of the ball and the position of the ball around the circumference do not effect the dynamics of the other variables. By assuming $a + r < R$, the system (1) is a smooth vector field in the four dimensional phase space $(\omega_1, \omega_2, \omega_3, \beta) \in \mathbb{R}^3 \times [-\pi, \pi)$. The system (1) itself is nonlinear, but further nonlinearities are caused by slipping of the ball or falling from the rim.

Analysis of stationary solutions

It can be determined that the system (1) has not an isolated stationary solution but a two-parametric family of stationary solutions in the form

$$\omega_1 \equiv 0, \qquad \omega_2 \equiv \frac{(1+j)r^2\omega_{30}^2 \sin\beta_0 - g\cos\beta_0(R-(a+r)\cos\beta_0)}{(1+j)(R-(a+r)\cos\beta_0)jr^2\omega_{30}\cos\beta_0}, \qquad \omega_3 \equiv \omega_{30}, \qquad \beta \equiv \beta_0. \tag{2}$$

That is, for any chosen tilt angle $\beta_0 \neq \pm\pi$ and circular angular velocity $\omega_{30} \neq 0$, there is a spinning angular velocity $\omega_{20}(\beta_0, \omega_{30})$ for which $(0, \omega_{20}(\beta_0, \omega_{30}), \omega_{30}, \beta_0)$ is a stationary solution of (1). Note that stationary solutions are not related mechanical equilibrium but the ball is rolling around the circumference of the rim at a constant tilt angle β_0 and it is rotating with constant circular and spinning angular velocities.

From (2), a subset of the stationary solutions are not realizable because it would require negative normal force between the ball and the rim. In the left panel of Figure 2, the shaded area shows the realizable stationary solutions with positive normal force and the white area correspond to the case when the ball falls down from the rim. Checking the condition of slipping from the Coulomb friction model leads to a similar but stricter condition than that of the falling of the ball. A further practical restriction is the limited kinetic energy of the ball from the shot.

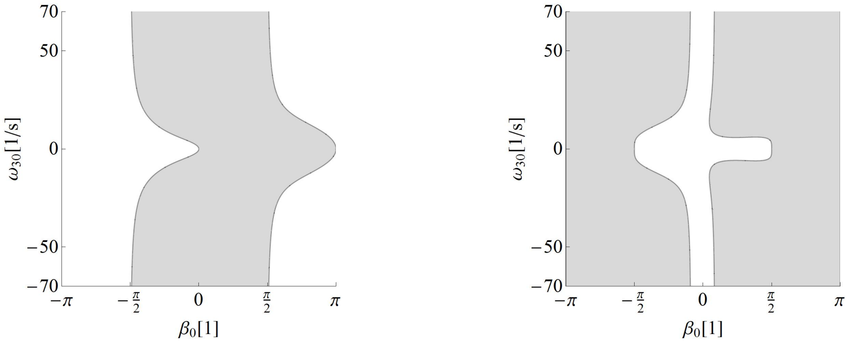

Figure 2: Conditions of the realisable stationary motions. Each point of the plane correspond to a stationary motion according to (2). Left panel: the shaded area shows the solutions which the ball does not fall from the rim. Right panel: the shaded area show the solutions which are stable. The numerical values of the parameters are the values of the standard basketball and basketball rim.

Linear stability analysis shows that some of the stationary solutions are unstable with a positive real eigenvalue. This corresponds to the white region in the right panel of Figure 2. For stationary solutions inside the shaded region of this diagram, there is a double zero eigenvalue and a pair of pure imaginary eigenvalues. Further analysis shows that these solutions are stable but not asymptotically stable, and the trajectories in the vicinity of these stable stationary points are periodic.

Determining falling of the ball by numeric simulation

By neglecting the dissipative effects, a general initial condition of (1) leads to one of the following types of trajectories:

1. the ball remains on the rim permanently,
2. the ball falls from the rim inside the basket,
3. the ball falls from the rim outside the basket,
4. the ball falls from the rim in such direction that it hits the rim again.

Points of the phase space can be categorised into these four categories. By performing numeric simulations, these four regions can be visualised, which results complicated patterns. If small energy dissipation is introduced into the model then the first category vanishes, and the ball can either fall outside or inside the basket. Solutions of the fourth category leads also falling inside or outside, but it needs the analysis of the impact between the ball and the rim. Solutions can be also categorised according to the time which is necessary for the ball to leave the rim. The solutions of the longest time are close to the stable stationary solutions when the ball is rolling around the rim several times. Sensitivity on initial conditions demonstrates the unpredictable behaviour of the ball in practical situations.

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