Passive vibration control with a bistable nonlinear absorber

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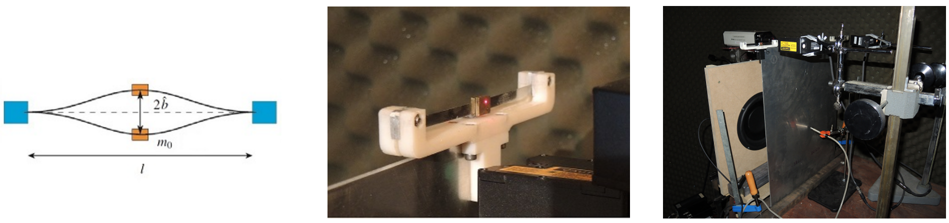
Summary. The goal of our research is to develop a nonlinear absorber that will be effective at low frequencies and low amplitudes of vibration, notably in the domain of vibroacoustics. To achieve this goal we developed the idea of a bistable absorber. This bistability provides a chaotic regime that leads to a particular mechanism of energy dissipation by the absorber. While this absorption is completely different from the well-studied targeted energy transfer (or energy pumping), the two types share some features such as the activation threshold and the functioning region. The experimental and numerical tests performed for our particular realization of a bistable absorber have shown the high efficiency of the absorber, its robustness and the way to control the functioning region.

Introduction

Following the fundamental work of Gendelman *et al* [1, 2] on the energy pumping, the use of nonlinear systems as absorbers got a lot of interest from both scientific and industrial communities. The general name that was given to this type of absorbers is "Nonlinear Energy Sink" or simply "NES". The NESs have found their applications in acoustics [3] and mechanics [4]. The main limitation for the NES in case of vibroacoustics is the existence of the triggering threshold for the energy pumping, that is usually to high for the applications.
The recent numerical and theoretical work of Romeo *et al* [5] have shown that a bistable configuration, that possesses both quadratic and cubic nonlinearity, shows a new approach that can pave the way for the NES to vibroacoustics. This type of nonlinearity provides a region in which NES dynamics becomes chaotic. In this regime the NES can effectively absorb the energy, but at the same time, since its dynamics is chaotic, it re-injects the energy back into the primary system (thus the word "Sink" is not more suitable). While this mechanism of the energy transfer is completely different from the one for the usual energy pumping, both of them share similar features: the triggering threshold and the limited functioning region.
To study the bistable NES efficiency in case of primary systems with more than one mode, we performed experimental and numerical study for our particular realization of a bistable absorber.

Absorber and its Modeling

To create a bistable system we started from a well-known system - clamped-clamped buckled beam. To amplify the forcing of its vibrations we added a mass that we glue on the beam. The support that clamps the beam was attached to a plate that represents the linear system. It is important to emphasize that the mid-span beam deflection of the beam was more than 50 times bigger than the thickness of the NES beam, while the mass that we added was two times bigger than the mass of the beam itself. The NES scheme and its mounting on the plate (our primary system) are shown in figures 1(a) and 1(b). The total mass of the NES (without support that attaches it to the plate) was 2.5g.



(a) NES scheme (b) NES mounted on a plate (c) Experimental installation.

Figure 1: The bistable NES

The experimental setup used to probe the NES efficiency is shown in figure 1(c). The 2kg aluminum plate was clamped from below and free from the other three sides. A loudspeaker excited the plate vibrations. The NES was attached to one of the upper edges of the plate. The reference excitation pressure was measured behind the loudspeaker close to its membrane. The plate and NES dynamics were traced by velocitymeters and displacement lasers.
It is easy to see that the difficulty for modeling of this system lays in the NES itself, while after the mode projection the description of the linear system and its coupling to the NES are straightforward .
To describe the NES we suppose that the mass is point-like and the beam is uniform: this allows us to use the equation for a clamped-clamped buckled beam as a base of our model. We also suppose that the boundary conditions are ideal. Under these approximations we can write the equation for a transverse planar vibrations of our NES [7]:

$$(m + M\delta(x - x_0))\frac{\partial^2\Phi}{\partial t^2} + EI\frac{\partial^4\Phi}{\partial x^4} + P\frac{\partial^2\Phi}{\partial x^2} + c\frac{\partial\Phi}{\partial t} = \frac{EA}{2l}\frac{\partial^2\Phi}{\partial x^2}\int_0^l \left(\frac{\partial\Phi}{\partial x}\right)^2 dx + F(x,t), \quad (1)$$

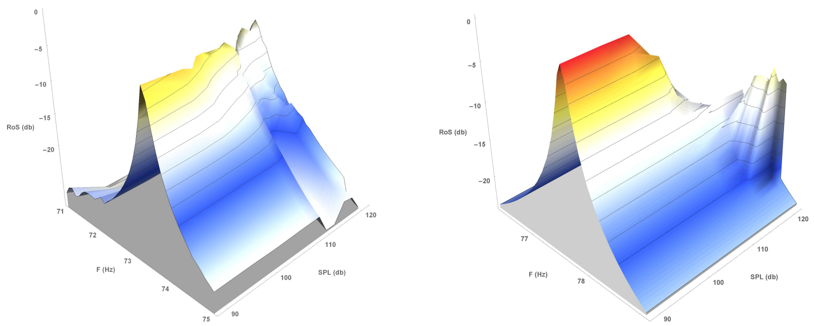
with the boundary conditions

$$\Phi = 0 \text{ and } \frac{\partial \Phi}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = l. \tag{2}$$

Here $\Phi(x,t)$ represents the transverse beam displacement as a function of coordinate along the beam x and time t, m is the mass per unit length, E is the Young's modulus, A and I are the area and the moment of inertia of the cross section, respectively, l is the length of the non-deformed beam, P is the axial load, c is the viscous damping coefficient, $F(x,t)$ is the external force. M is the added mass and x_0 is the point of its attachment. Beside the presence of the δ-part, it is the equation for a buckled clamped-clamped ideal beam (see [6]).
Since the buckling is very high and the added mass is comparable to the mass of the beam we could not make any direct simplifications in this equation, such as one-mode description. For these reasons, we first resolved analytically the linearized form of this equation to obtain the linear vibration modes of the NES beam around one of the two equilibrium positions. To perform simulations that include snap-through motion we applied Galerkin discretization procedure using the linear modes as the basis functions. Depending on the mass position we needed from 50 to 60 modes to obtain a good description of the complete NES dynamics.

Results and conclusions

In both cases, numerical and experimental, the loudspeaker provided a constant-amplitude monochromatic excitation around fourth plate resonance of 73Hz. Changing step by step the frequency and the amplitude we excited the system from the rest position and recorded signals after the disappearance of transients. The vibration response of the plate was characterized by a "response over signal" function $RoS = 20\log(V_{RMS}/SPL_{RMS})$, where V_{RMS} and SPL_{RMS} are, respectively, the root mean square of the plate velocity at the point of the NES attachment and the root mean square of the excitation sound pressure level recorded during 10 seconds of the forced plate vibrations. Figure 2 shows the resulting ridge surfaces.



(a) Experimental results (b) Results of the simulation

Figure 2: The ridge surfaces for the excitation around the fourth mode.

As we can see, above an excitation level of 100-110 dB we observe an attenuation of the plate vibration coupled to the NES. There is a discrepancy between the simulation and experiment for the response at low level of the excitation. This difference is due to the imperfections in the NES assembling that accidentally led to the Frahm absorber effect; this effect is obviously valid only in the linear regime. Nevertheless, the experiment, as well as the simulation, shows that the most important attenuation correlates with the chaotic dynamics where the NES changes the equilibrium position. This attenuation has the same value for the simulation and the experiment and can be as high as 10dB (see the summary for the ridge curves in Figure 3).

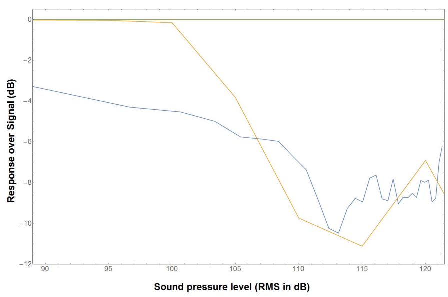
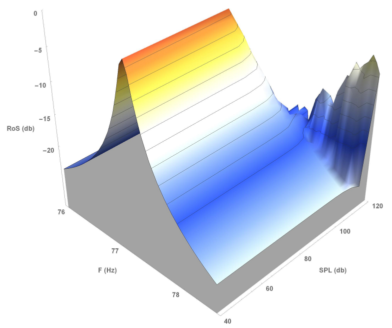
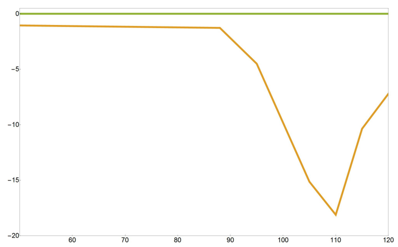


Figure 3: The ridge curves for the excitation around the fourth mode. Yellow line - simulation, blue line - experiment, Green line represents the reference level of a linear system with blocked NES.

The simulation shows that in the chaotic regime the energy is transferred to the NES and then re-injected back into the system, exciting the other modes of the plate. Further studies have shown that the triggering threshold and the functioning region can be controlled by the change of the buckling level and the mass position for the NES. Figure 4 shows the results of the simulation of a similar experiment, but with the mass of the NES that was displaced by $1/25$ of the beam length.



(a) Ridge surface



(b) Ridge curves: Yellow - free NES, Green - Blocked NES (reference)

Figure 4: The results of the simulation for the NES with the added mass that was displaced from the center.

While in the case of the energy pumping the higher limit of the NES efficiency region corresponds to the disappearance of the internal resonance, for the bistable NES it corresponds to a certain regularization of the NES motion. This regularization is represented by more regular changes of the equilibrium position of the NES (the emerging peaks in Figures 2 and 3 for the high excitation level).
As it can be seen from the nonlinearities of the equation for the bistable system, the usual energy pumping is also possible: at low amplitude excitation - due to the softening behavior, and at high amplitudes due to the hardening and the regularization of the NES motion. The merging of the regions of the energy pumping and the chaotic regime can be of a particular interest since it can be a way to prolong the "plateau" of the energy pumping.

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