Using a robust torus to control chaos in low density beams

M. C. de Sousa and I. L. Caldas
Instituto de Física, Universidade de São Paulo, São Paulo, Brazil

Summary. We investigate the dynamics of a relativistic low density beam moving in a uniform magnetic field and interacting with a stationary electrostatic wave. In the resonant islands of the system, the wave transfers energy to the particles and they are regularly accelerated. When we increase the wave amplitude, the system becomes chaotic and the islands used for particle acceleration are destroyed. To reduce the number of chaotic trajectories, we introduce a robust torus that controls chaos in phase space and restores resonant islands. We show that when the robust torus is present, it is possible to continue accelerating particles for higher values of the wave amplitude.

Introduction

Wave-particle interaction is present in many areas of Physics [1] and it is used in several applications for particle heating [1, 2] and particle acceleration [1, 3]. Particle acceleration occurs in the resonant islands of the system, and it is affected by the presence of chaos in phase space, which destroys regular islands [4, 5]. As an alternative to avoid this problem and regularize the system, we apply methods of control of chaos [6, 7, 8].
In this paper, we analyze the dynamics of a low density beam interacting with a uniform magnetic field and a stationary electrostatic wave. When wave and particles are in resonance, the wave transfers a great amount of energy to the particles and they are regularly accelerated [4, 5]. The amount of energy transferred to the particles is proportional to the wave amplitude. However, even for low values of the wave amplitude, the system becomes chaotic, regular islands are destroyed and it is no longer possible to regularly accelerate the particles.
We consider the main resonance of the system for which the initial energy of the particles may be close to their rest energy. To control chaos around the main resonance, we create a robust torus in phase space. We show that when the robust torus is properly located, it is highly effective in controlling chaos near the main resonance and, as a consequence, it improves the process of particle acceleration from low initial energies.

Description of the System

We analyze a low density beam composed by relativistic charged particles. For low density beams, the particles may be considered as test particles that do not interact with each other. In this case, the dynamics of the beam is determined by the behavior of individual particles.
Following Refs. [5, 8], we use a uniform magnetic field $\vec{B} = B_0\hat{z}$, with vector potential $\vec{A} = B_0 x\hat{y}$, to confine the particles. The beam also interacts with a stationary electrostatic wave given as a series of periodic pulses with wave vector $\vec{k} = k\hat{x}$, period T and amplitude $\epsilon/2$. Using dimensionless quantities, the Hamiltonian that describes the dynamics transverse to the magnetic field is written as [5, 8]

$$H(x, p_x, t) = \sqrt{1 + p_x^2 + x^2} + \frac{\epsilon}{2}\cos(kx)\sum_{n=-\infty}^{+\infty}\delta(t - nT). \tag{1}$$

We perform the canonical transformation $x = \sqrt{2I}\sin\theta$ and $p_x = \sqrt{2I}\cos\theta$, and we rewrite Hamiltonian (1) as

$$H(\theta, I, t) = \sqrt{1 + 2I} + \frac{\epsilon}{2}\cos(k\sqrt{2I}\sin\theta)\sum_{n=-\infty}^{+\infty}\delta(t - nT). \tag{2}$$

In the (θ, I) coordinates, it is easier to notice that the Hamiltonian of the system is integrable between two consecutive pulses of the wave.

Robust Torus

In the resonant regions of the system described by Hamiltonian (2), the wave transfers a considerable amount of energy to the particles and they are regularly accelerated. However, the system becomes chaotic even for low values of the wave amplitude, and the chaotic trajectories destroy resonant islands and affect particle acceleration.
To control chaos in the system, we create a robust torus in its phase space. The Hamiltonian that includes the robust torus is given by

$$H = \sqrt{1 + 2I} + \frac{\epsilon}{2}(I - I^*)^2\cos(k\sqrt{2I}\sin\theta)\sum_{n=-\infty}^{+\infty}\delta(t - nT), \tag{3}$$

where I^* indicates the position of the robust torus in phase space. The term $(I - I^*)^2$ in Hamiltonian (3) reduces the perturbation caused by the wave around I^*, and for $I = I^*$, the perturbation vanishes. It means that the robust torus is able to control chaos in a region close to $I = I^*$.

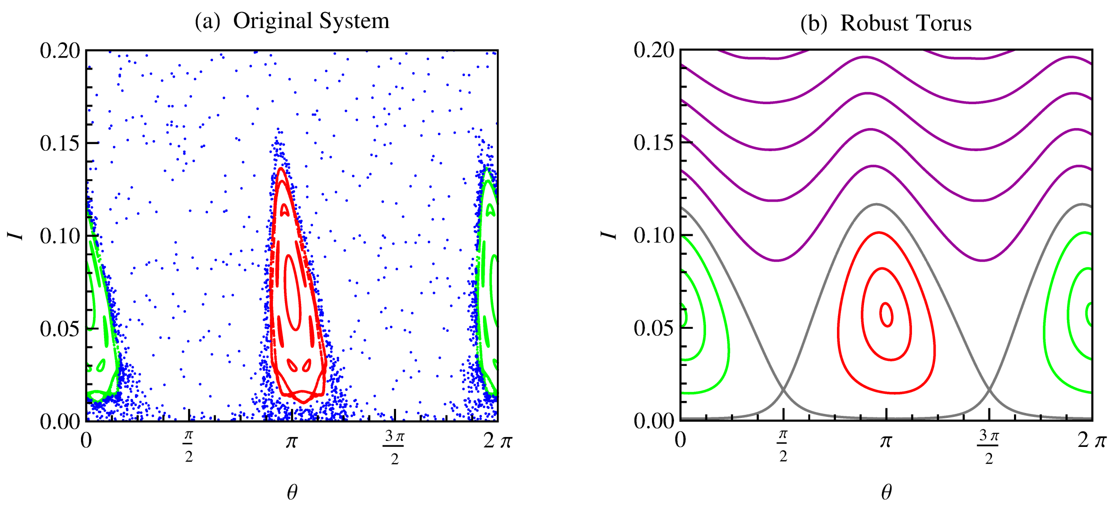

Figure 1: Phase space of the system (a) without the robust torus, and (b) with the robust torus.

Figure 1 shows the phase space of the system around its main resonance. In the pictures, the wave period is slightly larger than the cyclotron period ($T = 2\pi(1+1/15) > 2\pi$), and the main resonance is close to the axis $I = 0$, which corresponds to the rest energy of the particles. In Panel 1.(a), we show the phase space of the original system. Great part of the phase space is covered by chaotic trajectories (in blue). The most external trajectories of the resonant islands (in red and green) were destroyed by chaos and what remains from the islands is not suitable for particle acceleration.
In Panel 1.(b), we represent the phase space with a robust torus located at $I^* = 0.5$. We observe that the region around the main resonance is regularized by the robust torus. The resonant islands are restored and they may be used for particle acceleration once again. In this picture, the separatrix of the islands (in grey) almost touch the axis $I = 0$. Thus, the particles may be accelerated from initial energies very close to their rest energy and achieve 43% of the speed of light.

Conclusions

We analyzed a relativistic low density beam moving under the action of a uniform magnetic field and a stationary electrostatic wave. We considered the main resonance of the system and we observed that the presence of chaos in phase space destroys the regular resonant islands that are used to accelerate the particles of the beam. We added a robust torus to the system and we showed that it is able to control chaos in phase space, restoring the resonant islands and the process of particle acceleration from initial energies close to the rest energy of the particles.

Acknowledgments

We acknowledge financial support from the Brazilian scientific agencies: São Paulo Research Foundation (FAPESP) under Grants No. 2015/05186-0 and No. 2011/19296-1, Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (Capes).

References

[1] Shukla P.K. *et al.* (1986) Relativistic nonlinear effects in plasmas. *Phys. Rep.* **138**:1.

[2] Wang C.B., Wu C.S., Yoon P.H. (2006) Heating of ions by Alfvén waves via nonresonant interactions. *Phys. Rev. Lett.* **96**:125001.

[3] Edwards D.A., Syphers M.J. (2004) An introduction to the physics of high energy accelerators. Wiley-VCH, Weinheim.

[4] Pakter R., Corso G. (1995) Improving regular acceleration in the nonlinear interaction of particles and waves. *Phys. Plasmas* **2**:4312.

[5] de Sousa M.C. *et al.* (2010) Standard map in magnetized relativistic systems: Fixed points and regular acceleration. *Phys. Rev. E* **82**:026402.

[6] Ott E., Grebogi C., Yorke J.A. (1990) Controlling chaos. *Phys. Rev. Lett.* **64**:1196.

[7] Chandre C. *et al.* (2005) Channeling chaos by building barriers. *Phys. Rev. Lett.* **94**:074101.

[8] de Sousa M.C. *et al.* (2012) Controlling chaos in wave-particle interactions. *Phys. Rev. E* **86**:016217.