Friction Dependency of the Controllability of Rigid Bodies in Ideal Fluids

Sergey M. Ramodanov*, Alexey A. Kireenkov**
A.Blagonravov Institute of Machines Science of the Russian Academy of Sciences, Moscow, Russia
**A.Ishlinsky Institute for Problems in Mechanics of Russian Academy of Sciences, Moscow, Russia - Moscow Institute of Physics and Technology (State University), Dolgoprudny, Russia*

Summary. We study a mechanical system that consists of a 2D rigid body immersed in an unbounded volume of perfect fluid. The body is controlled by two internal point masses. The fluid's influence manifests itself through the added mass effect and dissipation (whose magnitude is small) described by the Rayleigh dissipation function. These assumptions allow for an analytical study of the problem using the small parameter method. Indeed, the unperturbed system (no dissipation) has, according to the Noether's theorem, a complete set of integrals of motion. These linear in velocities integrals can be expanded to the perturbed system in the form of series in the small parameter. For a fixed value of the integrals, the equations of motion reduce to a non-autonomous system of ordinary differential equations. The controllability of the system is then studied. The influence of dissipation on the controllability is outlined.

Problem Statement

Consider a 2D rigid body moving in an infinite immense of incompressible perfect fluid resting at infinity. There are no other external forces and torques acting on the body except for the hydrodynamic reactions, which manifest themselves through the added-mass effect and small dissipative force described by the Rayleigh dissipation function εR (ε is a small parameter). The body is controlled by two internal masses that move along circles. Their rates of rotation $\Omega_1(t)$ and $\Omega_2(t)$ are the control inputs for the system. The governing equations read:

$$\left(\frac{\partial T}{\partial u}\right)^{\cdot} - \omega\frac{\partial T}{\partial v} = -\varepsilon\frac{\partial R}{\partial u}, \quad \left(\frac{\partial T}{\partial v}\right)^{\cdot} + \omega\frac{\partial T}{\partial u} = -\varepsilon\frac{\partial R}{\partial v}, \qquad (1)$$
$$\left(\frac{\partial T}{\partial \omega}\right)^{\cdot} + u\frac{\partial T}{\partial v} - v\frac{\partial T}{\partial u} = -\varepsilon\frac{\partial R}{\partial \omega}$$

where T is the kinetic energy of the system, u, v and ω are the components of the absolute velocity of the body's center and the angular velocity of the body (Fig. 1a). Both T and R are quadratic forms in u, v and ω.

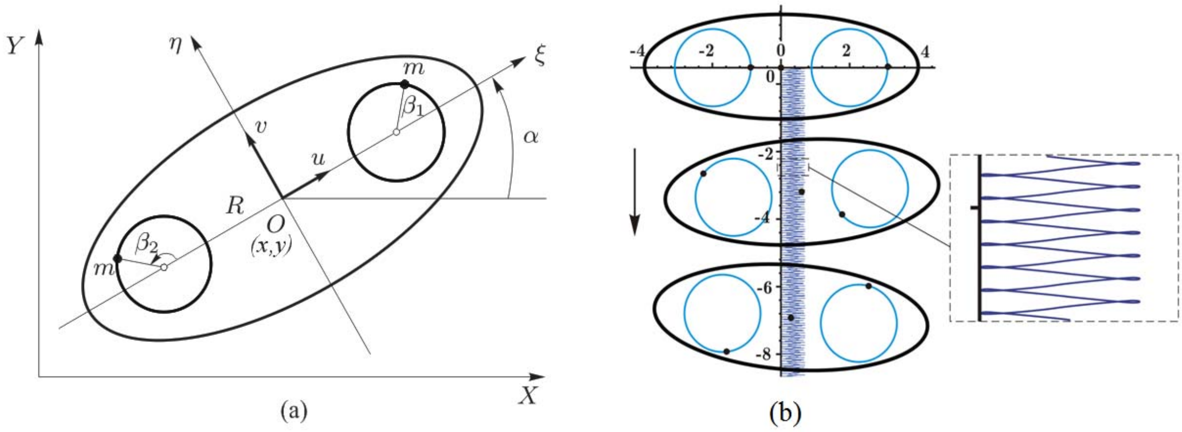

Figure 1.

No-friction case ($\varepsilon = 0$)

In this case, assuming the body to be initially at rest, equation (1), admit three Noether integrals of motion:

$$P_1 = \frac{\partial T}{\partial \dot{x}} = \frac{\partial T}{\partial u}\cos\alpha - \frac{\partial T}{\partial v}\sin\alpha = c_1, \quad P_2 = \frac{\partial T}{\partial \dot{y}} = \frac{\partial T}{\partial u}\sin\alpha + \frac{\partial T}{\partial v}\cos\alpha = c_2$$
$$P_3 = x\frac{\partial T}{\partial \dot{y}} - y\frac{\partial T}{\partial \dot{x}} + \frac{\partial T}{\partial \omega} = c_3 \tag{2}$$

which, being solved for u, v and ω, yield what is known as control-affine system:

$$(\dot{x}, \dot{y}, \dot{\alpha}, \dot{\beta}_1, \dot{\beta}_2)^T = \Omega_1\mathbf{V}_1 + \Omega_2\mathbf{V}_2$$
$$\mathbf{V}_1^T = (X_1(\alpha, \beta_1, \beta_2), Y_1(\alpha, \beta_1, \beta_2), \Phi_1(\beta_1, \beta_2), 1, 0), \tag{3}$$
$$\mathbf{V}_2^T = (X_2(\alpha, \beta_1, \beta_2), Y_2(\alpha, \beta_1, \beta_2), \Phi_2(\beta_1, \beta_2), 0, 1)$$

Provided that the body is hydrodynamically non-symmetric (meaning that the body is essentially prolate), the system satisfies the conditions of the Rashevsky-Chow [1] theorem and thus is fully controllable [2]. Some effective gates are found to prove that the body can be driven to any desired position. For example, when the masses go synchronously in opposite directions the motion of the body's center is practically rectilinear (Fig.1b).

Dissipative case ($\varepsilon \neq 0$)

This case presents an intriguing interplay between the viscous friction and the friction proportional to the acceleration (added-mass effect). Now the body can obviously propel itself even with its added masses being equal [3]. In particular, moving the masses synchronously in opposite directions no longer produces the path depicted in Fig. 1b (the actual path will gradually deflect to the right). Necessary corrections to the functions $\beta_1(t)$ and $\beta_2(t)$ for the path to remain almost rectilinear are evaluated.

Acknowledgements

This research was financially supported by the Russian Foundation for Basic Researches under grant No. 16-01-00809-a.

References

[1] Sussmann H.J. (1973) Orbits of families of vector fields and integrability of distributions. *Trans. Amer. Math. Soc* **180**: 171–188.
[2] Ramodanov S.M., Tenenev V.A. (2015) The motion of a two-dimensional body, controlled by two moving internal masses, in an ideal fluid. *JAMM* **79(4)**: 325–33.
[3] Chernous'ko, F.L. Optimal control of the motion of a multilink system in a resistive medium. *JAMM* **76(3)**: 255-67.