Borehole Spiraling as Limit Cycle of Directionally Unstable Drilling Systems

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Summary. Borehole spiraling is shown to represent a limit cycle of the dynamics of unstable directional drilling systems. The delay differential equations governing the borehole propagation result from considerations involving a bit/rock-interaction law, kinematic relationships that describe the local borehole geometry in relation to the bit penetration, and a beam model of the bottom-hole assembly (BHA). The geometrical constraints, arising from the stabilizers conforming to the borehole geometry, affect the forces at the bit and thus the drilling direction through the forced deformation of the BHA. These constraints are the sources of the spatial delays in the borehole propagation equations. Saturation of the bit tilt with increasing transverse force at the bit leads to the establishment of a limit cycle of the bit tilt oscillations when the directional drilling system is unstable, provided that the transverse force remains under a certain threshold.

Introduction

Borehole spiraling is a common drilling dysfunction that is still not easily identified in real time without state-of-the-art downhole measurement devices. The observed spiraling or micro-tortuosity of the borehole is the expression of self-excited oscillations of the bit tilt, which arise from the geometric influence of the stabilizers on the drilling direction at the bit as they interact with the borehole [1, 2, 3]. Indeed, as the BHA is constrained by the stabilizers to conform to the borehole, the resulting geometric constraints tend, under certain conditions, to progressively amplify at the bit local perturbations in the borehole geometry. Once a limit cycle is established, because of emergent nonlinearities in the response of the system, it is maintained by the fluctuating lateral force at the bit. These fluctuations in the drilling direction of the bit actually betray variations of the bit tilt, as the orientation of the bit axis remains almost constant. This paper outlines a mathematical model of the directional drilling system, which shows the existence of a limit cycle corresponding to the formation of a regular spiraling of the borehole when the system is unstable.

Borehole Propagation Equations

The three-dimensional model of borehole propagation is built upon three components: a bit/rock interaction law, a beam model of the BHA, and kinematic relationships at the bit. The bit/rock interaction law relates the force and moment acting on the bit to its penetration into the rock, while the BHA model expresses these force and moment as a function of the external loads applied on the BHA and of the constraints imposed by the borehole geometry via the stabilizers. Finally, the kinematic relationships connect the bit motion to the local borehole geometry (Fig. 1). Combination of these three elements lead to two coupled delay differential equations (DDE) in terms of azimuth Φ and inclination Θ of the borehole, with the spatial delays corresponding to the positions of the stabilizers relative to the bit. The general scaled form of these two DDE's are

$$\epsilon\Xi'(\xi) = F_{\Xi}\left(\Theta(\xi), \Phi(\xi), \Theta(\xi-\xi_i), \Phi(\xi-\xi_i), \langle\Theta\rangle_i, \langle\Phi\rangle_i, \mathbf{\Gamma}, \mathbf{\Gamma}', \Upsilon\right) \tag{1}$$

where Ξ is either Θ or Φ, variable ξ is the borehole length, ξ_i, $i = 1, n$ denote the positions of the stabilizers behind the bit which is located at ξ, $\langle\Xi\rangle_i$ is the average inclination or azimuth of the BHA segment defined by the $i-1$ and i stabilizers, $\mathbf{\Gamma}$ represents the force applied by a rotary steerable system (RSS) on the BHA, and $Upsilon$ is a number related to the weight of the BHA. The small parameter ϵ reflects the design of the bit. Fundamentally, these equations capture the geometric feedback brought by the stabilizers, which is the root cause of borehole spiraling. Under constant loading, these equations show the existence of quasi-stationary ("long" borehole asymptotics) and stationary solutions.

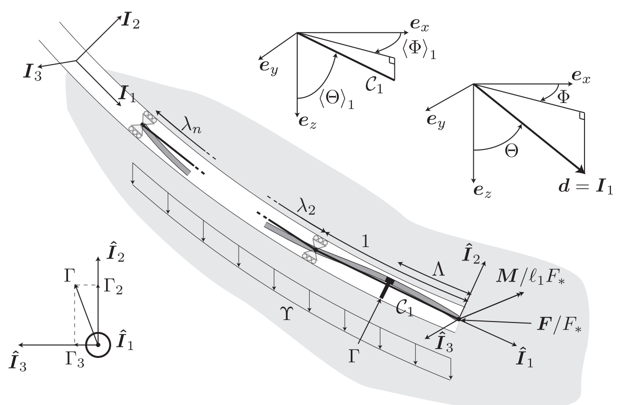


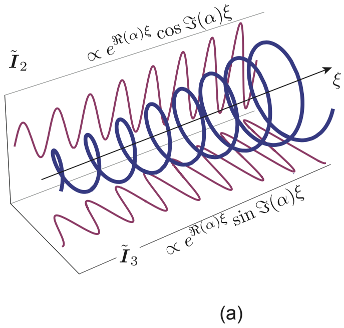
Figure 1: Model of the BHA and representation of the force and moment at the bit in the reference system [2].

Stability of the Directional Drilling System

The stability of the quasi-stationary solutions is established by determining whether pertubations $\delta\Xi$ and $\delta\Delta$ (where pseudo-azimuth Δ is defined as $(\Phi - \Phi_o)\sin\Theta_o$ with the subscript o denoting reference values) grow or decay exponentially. The linearized equations governing the propagation of perturbations $\delta\Xi$ and $\delta\Delta$ are of the form

$$\epsilon\delta\Xi'(\xi) = \mathcal{A}_{\Xi\Theta}\Theta(\xi)+\sum_{i=1}^{n}\mathcal{B}_{\Xi\Theta,i}\delta\Theta(\xi-\xi_i)+\sum_{i=1}^{n}\mathcal{C}_{\Xi\Theta,i}\left\langle\delta\Theta\right\rangle_i(\xi)+\mathcal{A}_{\delta\Xi\Delta}\delta\Delta(\xi)+\sum_{i=1}^{n}\mathcal{B}_{\Xi\Delta,i}\delta\Delta(\xi-\xi_i)+\sum_{i=1}^{n}\mathcal{C}_{\Xi\Delta,i}\left\langle\delta\Delta\right\rangle_i(\xi) \tag{2}$$

where $\delta\Xi$ is either $\delta\Theta$ or $\delta\Delta$ and where the coefficients $\mathcal{J} = \mathcal{A},\mathcal{B},\mathcal{C}$ depend on the distance between the stabilizers (the spatial delays), on the bit walk that measures the non-coaxiality between the lateral force and the lateral penetration at the bit, and on a dimensionless group $\eta\Pi$ function of the properties of the bit and of the BHA, the bit wear, and the weight on bit. The coefficients satisfy some symmetry, namely $\mathcal{J}_{\Theta\Theta} = \mathcal{J}_{\Delta\Delta}$ and $\mathcal{J}_{\Theta\Delta} = -\mathcal{J}_{\Delta\Theta}$, noting that the coupling between the two equations (2) disappears in the absence of a bit walk. A linear stability analysis of (2) yields two important results: namely, (i) the drilling system is directionally unstable, when $\eta\Pi$ is smaller than a critical value $\eta\Pi|_s$ —a function of the positions of the stabilizers on the BHA and of the bit walk; (ii) the unstable perturbation modes are helices with a pitch about equal to the distance between the bit and the first stabilizer (Fig. 2).



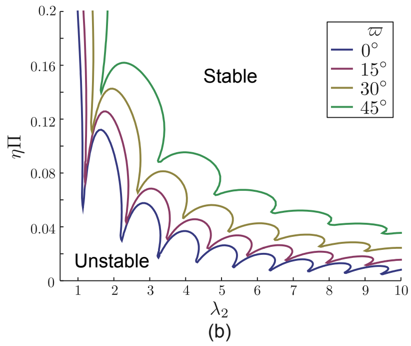
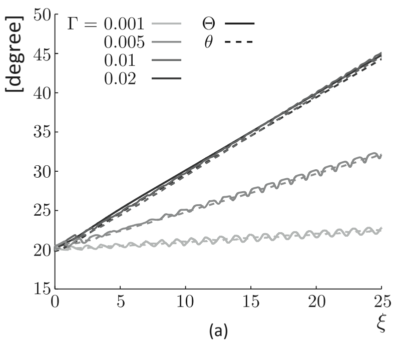


Figure 2: (a) conic helix representing the unstable perturbation mode of a borehole. (b) Bifurcation diagram for a two-stabilizers drilling system showing the variation of critical value $\eta\Pi|_s$ with relative distance λ_2 between the first and the second stabilizer [2].

Limit Cycles

Accounting for a saturation of the tilt introduces an essential nonlinearity in the propagation equations. Numerical simulation show the establishment of a limit cycle, which almost perfectly corresponds to a spiral in the absence of a RSS or if the RSS force Γ is small enough. Figure 2 illustrates the critical influence of the lateral force and the tilt saturation on the shape of the limit cycle, on the limitation of the achievable steering capacities, and on the drift out of the vertical plane.



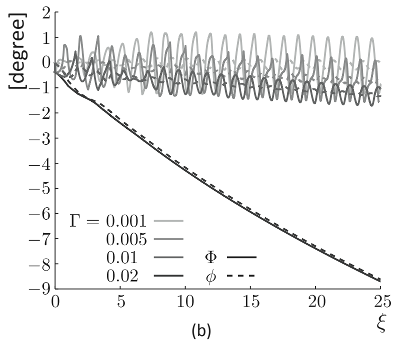


Figure 3: Influence of RSS force Γ on the limit cycle and the achievable curvature: (a) inclination and (b) azimuth (dashed line for the inclination and azimuth of the bit.

References

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