Model reduction for mercury porosimetry: invasion percolation on regular, exotic and random networks

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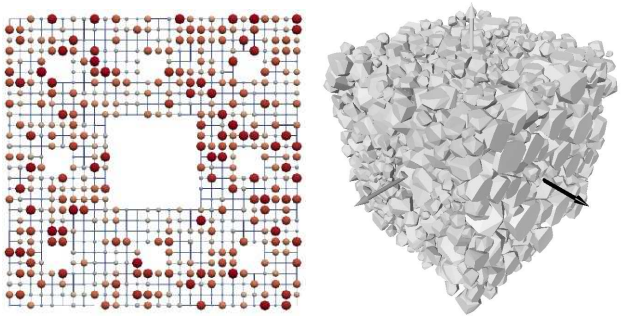
Summary. We study non-wetting liquid propagation in porous media. In particular, our goal is to simulate mercury porosimetry which is a measurement technique for determining the pore size distribution of rock samples. To computationally represent the propagation of mercury in porous medium we developed a percolation ("porcolation") model. We carried out simulations on different types of networks, including deterministic networks (simple and exotic lattices) and random networks with prescribed degree distribution (which represent the pore network of rock samples). The porcolation simulations give a better estimation of the pore size distribution.

Introduction

Mercury porosimetry is used primarily in the oil industry to determine the porosity and pore size distribution of rock samples. The measurement is carried out in a vacuum chamber filled up with mercury. The clean sample (dried and free of contamination) is immersed in mercury which is forced into the sample with increasing pressure. The volume of injected mercury is measured during the process. The result is the saturation S (the fraction of pore volume filled up with mercury) as the function of the applied pressure p. The size ρ of the smallest pore which can be entered by mercury is related to the applied pressure p by the Washburn-equation $p = -\frac{2\gamma\cos(\theta)}{\rho}$. Here γ is the surface tension and θ is the contact angle. In practice, based on this equation the pore size distribution of the sample is derived from the saturation curve, although it is known that the aquired PSD is incorrect [1]. The main problem is that there are larger pores which can only be accessed through smaller ones. Thus these pores are only occupied by mercury if the applied pressure is high enough (this is known as the shielding mechanism). Percolation theory was developed for a number of different applications including the modeling of flow in porous medium. The pore structure is modeled as a network of interconnected pores. Wilkinson and Willemsen [2] introduced the invasion percolation (IP) model which simulates the propagation of a non-wetting liquid in porous media. In IP a starting set of occupied pores is designated. The liquid interface consists of the unoccupied pores neighboring of occupied pores. An invasion resistance r is assigned to each pore. In one simulation step the liquid occupies the interfacial pore having the smallest invasion resistance. Porcolation (PORosimetry perCOLATION) is a modification of IP. Here the liquid propagation is driven by the external pressure p. Only those pores with $r \leq p$ are occupied to which the liquid finds its way through a path of pores from the starting set.

Porcolation simulations

The porcolation model was tested on different types of regular networks including simple ones (square and cubic lattices) and exotic networks (Sierpiński triangle and carpet, see Figure 1(a)). Figure 1(b) shows a virtual rock sample, which we modeled as a random network with prescribed degree distribution [3].



(a) (b)

Figure 1: (a) The Sierpiński-carpet type pore network model. (b) Isometric view of a virtual rock sample.

The saturation curves for the square and the cubic lattices are shown in Figure 2(a). Results suggest that the saturation curve depends on the network type and the invasion resistance distribution of the network. Figure 2(b) shows saturation curves for Sierpiński-carpets. These networks were created from the square lattice by deleting pores and connections. The higher the number of iterations is, the more pores and connections were deleted. In Figure 2(c) the experimental and simulated saturation curves are shown for two rock samples.
We found that porcolation can be treated as an input-output mapping $\mathcal{P}$ which results in a specific saturation curve $S(r)$ [4] (the saturation can be also interpreted as the function of the invasion resistance r, since it is a pressure type quantity) for a given cumulative invasion resistance distribution $\phi(r)$ of the pores, i.e.

$$\mathcal{P}(\phi(r)) = S(r). \tag{1}$$

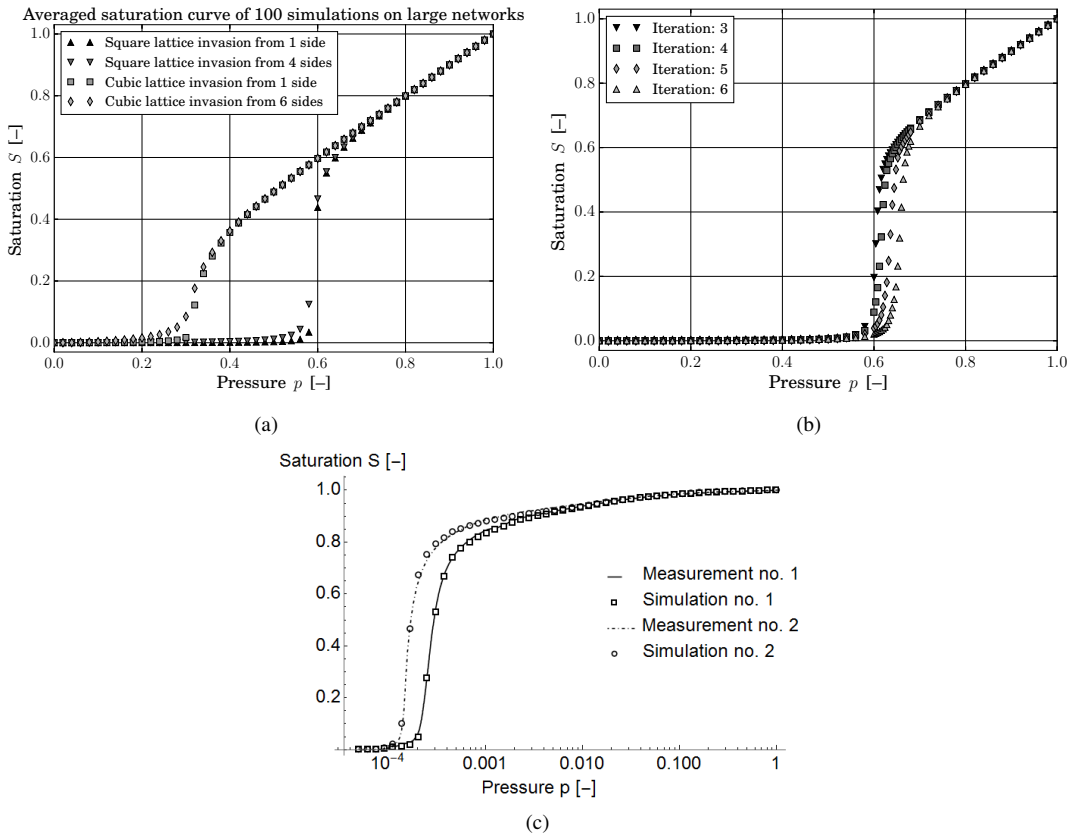


Figure 2: (a) Saturation curves for different lattice types and starting sets. (b) Saturation curves for Sierpiński-carpets with different sizes. (c) Porosimetry measurement results and simulated saturation curves on random networks with corrected invasion resistance distribution.

The mapping $\mathcal{P}$ only depends on the topology of the network. For a given input-output pair this mapping can be determined by the Inverse Transform Method [5]. Once $\mathcal{P}$ is known, the saturation curve $S(r)$ can be predicted from $\phi(r)$. Since both $\phi(r)$ and $S(r)$ are monotonously increasing, $\mathcal{P}$ is invertible. Consequently, the inverse problem

$$\mathcal{P}^{-1}(S(r)) = \phi(r). \tag{2}$$

can also be solved. The experimental saturation curve of the rock sample was used as the input invasion resistance distribution for the generated pore networks. Porcolation simulations were carried out on the generated networks to determine $\mathcal{P}$ and $\mathcal{P}^{-1}$. Using Eq. (2) we determined the corrected cumulative invasion resistance distribution of the pore network. Invasion resistance r is a pressure type quantity, thus the corresponding pore size ρ is calculated from the Washburn-equation. We obtained accurate pore size distributions.

Conclusions

Our goal was to find a more accurate pore size distribution of a given rock sample with simulation tools using its experimental saturation curve and a network model for the pore structure. The propagation of mercury is modeled by porcolation (an access-limited IP model). Simulations were carried out on different types of lattices and on random networks representing the pore structure of rock samples. We proposed a method to calculate the mapping between the input cumulative invasion resistance distribution and the simulated saturation curve. The inverse of this mapping was used to determine the correct cumulative invasion resistance distribution of the virtual rock samples. The method was tested on different types of networks and proved to be reliable.

References

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