Observations of Vibratory Force Phenomena

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Abstract: This paper demonstrates the effects that vibratory forces have on several vibrating objects. The forces that result from vibration may cause a change in position in some components of certain systems. The oldest known example is Huygen's clock pendulums. The authors have studied some systems that operate in such a way that they are able to compensate for the dynamic forces, change the position of equilibrium, alter it back and forth between static and unstable, and decrease the coefficient of friction. By controlling the components' vibration, it is possible to obtain a desired trajectory of motion of small elements.

1. Introduction

Vibratory forces are found in many systems in which some element has periodic motion. They are also found in everyday life whether we notice them or not, such as human locomotion. In general, animal locomotion results from the periodic movement of limbs. Legged mammals change position by moving their appendages; limbs work as pendulums and generate propulsion force. Terrestrial locomotion can take the form of walking, running, hopping, dragging, or crawling.

Limbless creatures operate under a different principle of locomotion, using their body to generate propulsive force - e.g. snakes and earthworms drag themselves across the ground by alternately scrunching up parts of their body and relaxing other parts. Pythons and boa constrictors have large scales on their underside which push backwards and downwards rhythmically in their rectilinear motion. Some animals (e.g. earthworms) burrow through solids such as soil using peristalsis. Undulation allows fish and other marine animals to generate thrust and some use fins to control direction. Very small water animals, e.g. paramecium, use their cilia to move. Volant animals must generate lift to ascend and remain airborne. One way to achieve this is by flapping wings, their periodic motion through the air generating an upward lift force. Fig. 1 presents some examples of locomotion as a result of vibratory forces.

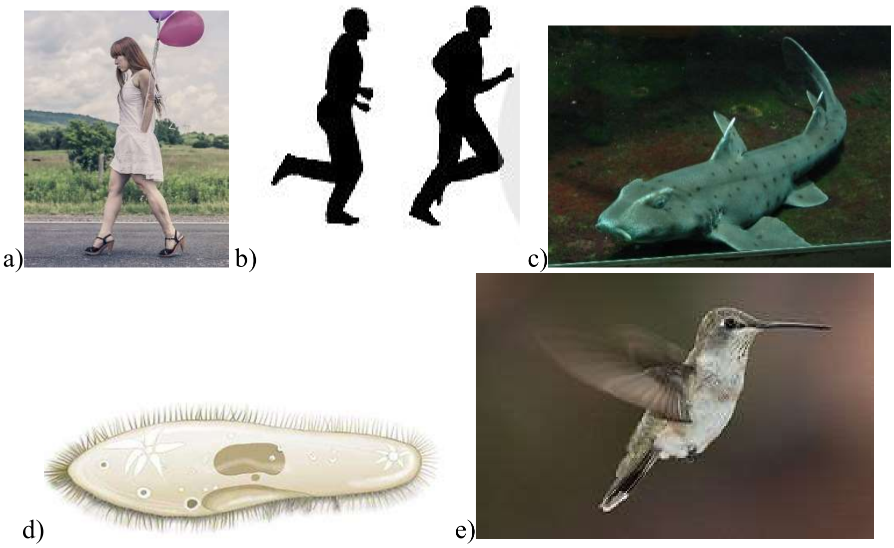


Fig.1. Examples of (a) walking, (b) running, (c) swimming, (d) paramecium and (e) humming bird

Forms of animal locomotion have inspired the design of several robots, several of which have been constructed successfully. In practice, such robots have more restrictions and limitations than do animals, depending on their intended use [1].

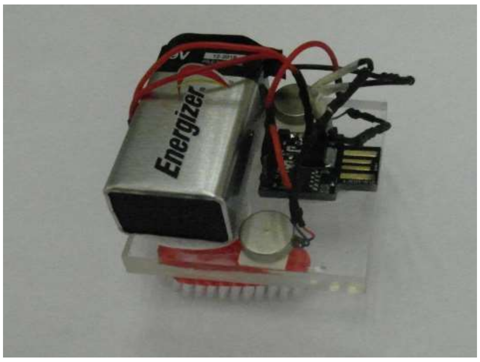


Fig.2. Vibratory robot with two disc motors

Synchronization is a common phenomenon in physical and biological systems [2, 3]. For example, swarms of fireflies gather in the evening at the same tree and start to flash chaotically, but after a while, they begin to flash in synchrony. Another example is the synchronization of clapping in audiences. Synchronization is the process in which two or more systems interact with each other and come to move together. It is commonly observed to occur between oscillators. The small motion of the base couples the pendulums, causing synchronization. One of the earliest phenomena was described by C. Huygens – the behavior of two pendulum clocks which were slightly "out of sync" mounted next to each other on a wooden board [4]. After a brief period, they began to move in synchrony, swinging in opposite directions. The vibrations of the common base generated by the pendulums are transmitted between them, generating vibratory forces which couple their behavior. Figure 1 demonstrates the pendulums moving synchronously and asynchronously.

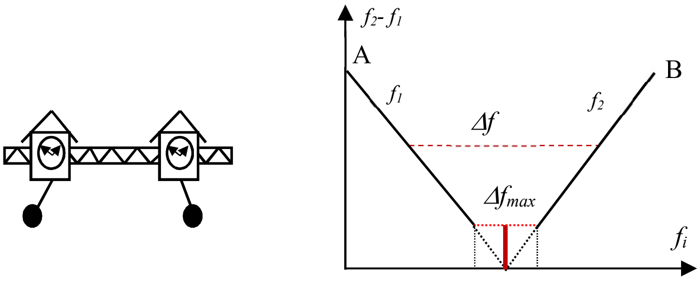


Fig.3. Pendulum clocks and their frequencies of motion

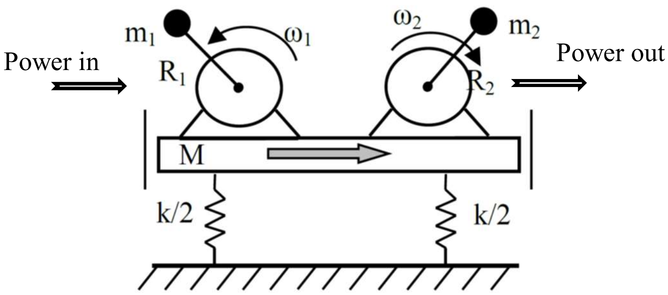


Fig. 4. Transmission of power between two rotors

Z. Engel demonstrated a transmission of the power between two rotors [5, 6]. Two unbalanced rotors on the same base started to move in synchrony with each other if the initial difference between their speeds was minor. It was possible to generate electricity from one rotor while the other was driven by an electric motor.

This article will discuss some examples in which vibratory forces play an important role and the ways they change the properties of some dynamic systems.

1. **Pendulum with pivot vibration** [Ref. 7]

Pivot O of the pendulum vibrates in harmonic way $z=z_o\sin(\Omega t)$ in the direction defined by angle β – Fig.5. The Mathieu equation defines the angular vibration of the pendulum if the pivot vibrates with frequency Ω. The natural frequency of the pendulum without pivot vibration is defined as $\omega_o = \sqrt{mge/B}$, where e defines the position of the mass center and B is the mass moment of inertia.

The vibration of the pendulum is as follows:

$$B\ddot{\psi} + c\dot{\psi} = -mge\sin\psi + mA\Omega^2 \sin\Omega t(\cos\beta\sin\psi - \sin\beta\cos\psi) = \\ = F(\psi) + P(\psi,\Omega t) \quad (1)$$

where B is mass moment of inertia of the pendulum, m is its mass, OC=e is the position of the mass center, A, Ω, β are the parameters of the pivot vibration, and two moments $F(\psi) = -mge\sin\psi$, and

$$P(\psi,\Omega t) = mAe\Omega^2 \sin(\psi - \beta)\sin\Omega t \; . \quad (2)$$

Moment $F(\psi)$ generates vibration $\alpha(t)$ with low frequency and moment $P(\psi,\Omega)$ generates vibration $\theta(\Omega t)$ with high frequency.

$$\psi(t,\Omega) \cong \alpha_o + \alpha(t) + \theta(\Omega t) \quad (3)$$

where α_o is the new position of equilibrium.

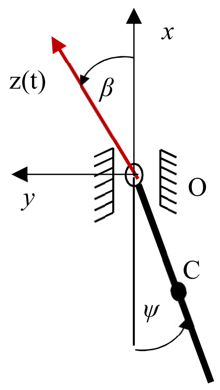


Fig.5. Pendulum with pivot vibration

The component vibrations are defined as follows:

$$B\ddot{\alpha} + c\dot{\alpha} = F(\alpha_o + \alpha + \Theta) = F(\alpha_o + \alpha) + P(\alpha_o + \alpha)\,, \quad (4)$$

$$B\ddot{\Theta} + c\ddot{\Theta} = P(\alpha,\Theta,\Omega t)\;. \quad (5)$$

Moment $P(\alpha, \theta, \Omega)$ can be put in Taylor series with respect θ and neglect the terms of higher order of θ

$$P(\alpha,\Theta,\Omega t) \cong P'(\alpha,\Omega t) + \frac{\partial P}{\partial \Theta}\Theta\,, \quad (6)$$

where

$$P'(\alpha,\Omega t) \cong meA\Omega^2 \sin(\alpha_o + \alpha - \beta)\sin(\Omega t)\,, \quad (7)$$

$$\frac{\partial P}{\partial \Theta} = meA\Omega^2 \cos(\alpha_o + \alpha - \beta)\sin(\Omega t)\;.$$

The small vibration with respect to the position of equilibrium is defined as

$$B\ddot{\Theta} + c\dot{\Theta} + k\theta = meA^2 \sin(\alpha_o + \alpha - \beta)\sin\Omega t\;. \quad (8)$$

The solution of Eq. (8) has a form

$$\Theta(\Omega t) = \Theta_o \sin(\alpha_o + \alpha - \beta)\sin(\Omega t - \varphi)\,, \quad (9)$$

where c is damping, $k = -meA\Omega^2 \cos(\alpha_o + \alpha - \beta)\sin(\Omega t)$.

For $\Omega >> \omega_o$, where ω_o the natural frequency, and small damping the amplitude of vibration Θ_o and phase angle φ can be taken as

$$\varphi \cong \pi, \qquad \Theta_o \cong \frac{meA}{B_o}, \qquad \Theta(\omega t) = -\Theta_o \sin(\alpha_o + \alpha - \beta)\sin(\Omega t) \quad (10)$$

The slow vibration $\alpha(t)$ depends on the average moment $\overline{P}(\psi,\Omega t) = \frac{1}{T}\int_0^T \frac{\partial P}{\partial \theta}\theta dt$ which has a form

$$\overline{P} = 0.5meA\Omega^2\Theta_o \sin 2(\alpha_o + \alpha - \beta)\int_0^T \sin^2(\Omega t)dt = 0.25meA\Omega^2\Theta_o \sin 2(\alpha_o + \alpha - \beta)\,. \quad (11)$$

The total moment acting on the pendulum from low and high frequency vibrations

$$M(\alpha_o,\alpha)=F(\alpha_o,\alpha)+\overline{P}(\alpha_o,\alpha)=-mge\sin(\alpha_o+\alpha)-0.25me\Omega^2 A\theta_o\sin(2\alpha_o+2\alpha-2\beta)\,. \qquad (12)$$

The low vibration $\alpha(t)$ is a function of the moment from the gravitational force and vibratory moment $\overline{P}$. The equation of vibration $\alpha(t)$ near the position of the equilibrium α_o takes a form

$$B_o\ddot{\alpha}+c\dot{\alpha}+me(g\cos\alpha_o+0.5\Omega^2 A\Theta_o\cos 2(\alpha_o-\beta))\alpha=$$
$$=-mge\sin(\alpha_o)-0.25me\Omega^2 A\Theta_o\sin(2\alpha_o-2\beta)\,. \qquad (13)$$

Eq. 12 defines the position of dynamic equilibrium and the Eq. 13 the natural frequency of vibration near this position

$$M(\alpha_o)=-meg\sin\alpha_o-.25me\omega^2 A\Theta_o\sin(2\alpha_o-2\beta)=0\ . \qquad (14)$$

$$\omega^*=\sqrt{meg(\cos\alpha_o+0.5\Theta_o(\Omega^2 A/g)\cos 2(\alpha_o-\beta))/B_o}\ , \qquad (15)$$

Frequency ω^* depends on amplitude A and frequency Ω.
From Eqs. 10 - 14 it can be concluded that for β=0, the position of equilibrium may be α_o=0° or π depending on frequency Ω and the amplitude of vibration θ_o near the upper position of equilibrium equals zero. Fig.6 presents the change of natural frequency and the form of vibration $\alpha(t)+\theta(t)$.

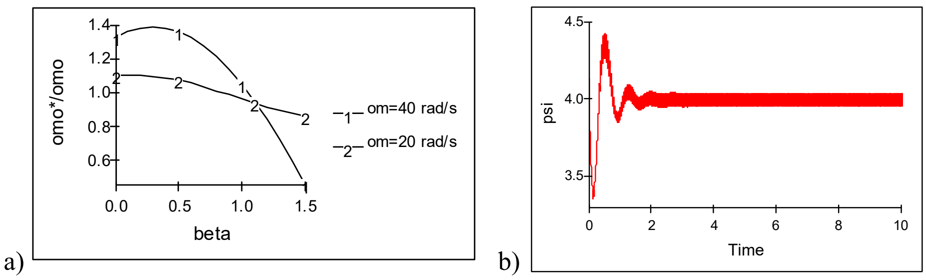


Fig.6. Change of natural frequency of the pendulum for lower position and vibration $\psi(t)$ of the pendulum for upper position if $A=16\ mm$, Ω=150 rad/s, β=30°

The vibration of the pendulum has two components; α(t) with frequency ω^* and the fast vibration $\theta(t)$ with frequency Ω - Fig. 6.

The position of equilibrium α_o is stable if $\left.\dfrac{\partial M}{\partial\alpha}\right|_{\alpha=\alpha_o}<0$ (16)

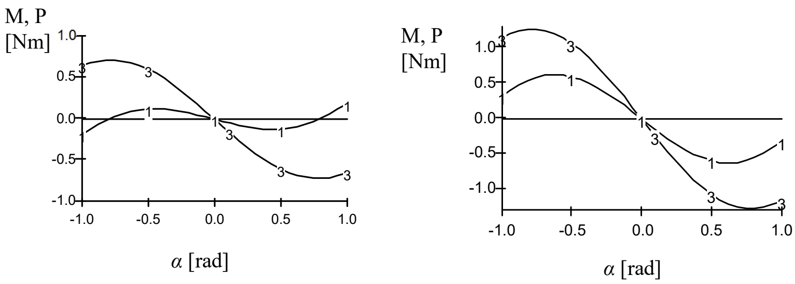


Fig.7. Total moment M (1) and vibratory moment P (3)
if Ω=150 (a), 200 rad/s (b) and β=0, $\alpha_o=\pi$

Fig.7 shows that the position of equilibrium is stable – the derivative of the resultant moment is negative.

As demonstrated, via the action of the vibration, the pendulum can change its position of equilibrium; the upper position which is statically unstable can be changed to dynamically stable and vice versa.

2. Increasing the speed of an unbalanced rotor.

The rotor with high imbalance has some properties which are more easily explained if the vibratory moment is defined. The imbalance mR is driven by a DC motor whose characteristic is $T_m = T_o - p\dot{\psi}$ - Fig.8,
where T_o, p are constants of the motor and $\dot{\psi}$ is its spin velocity.
The vibration of mass M and the angular motion of the imbalance are governed by the following equations:

$$M\ddot{x} + c\dot{x} + kx = me\,\dot{\psi}^2\cos\psi + me\,\ddot{\psi}\sin\psi\ , \qquad (17)$$
$$B\ddot{\psi} = T_m + me\ddot{x}\sin\psi = T_m + T_v(\alpha)\ .$$

When the imbalance spins with an angular velocity ω, then its angle of rotation is defined as

$$\psi \cong \omega t + \alpha \qquad \dot{\psi} \cong \omega\ .$$

The solution of Eq. 17 has a form

$$x \cong a_x \sin(\omega t + \alpha - \varphi_x)\ . \qquad (18)$$

where a_x is an amplitude of vibration and φ is a shift angle.

The vibratory moment

$$\overline{T}_v \cong \frac{1}{T}\int_0^T me\ddot{x}\sin\alpha \cdot dt = -0.5m\omega^2 e a_x \sin\varphi_x]\ . \qquad (19)$$

The vibratory moment always acts as a braking moment in the opposite direction of the rotor rotation, and its maximum occurs at the resonance velocity. The steady motion of the imbalance occurs at the angular velocity ω_1 when

$$T_m(\omega_1) + T_v(\omega_1) = 0\,, \quad \text{and it is dynamically stable if} \quad \left.\frac{\partial T_m}{\partial \omega}\right|_{\omega_1} - \left.\frac{\partial T_v}{\partial \omega}\right|_{\omega_1} < 0 \qquad (20)$$

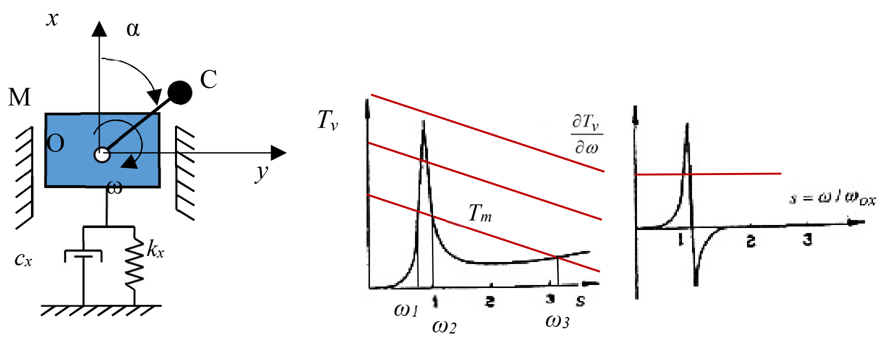
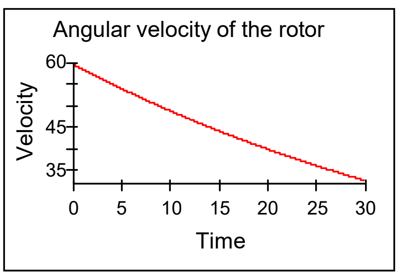


Fig.8. Unbalanced rotor and its dynamic characteristics

If the vibratory moment is larger than the torque of the motor, the rotor cannot move to over-critical range of velocity. For the highest characteristic of the motor (Fig. 8) increasing the motor velocity is not an issue, except with a rapid change of the rotor angular velocity ω at the resonance. For other characteristics $(T_m)_{max} < (T_v)_{max}$ there is an equilibrium velocity close to the natural frequency and the system cannot move to higher velocities.
When the motor is switched off, the velocity decreases rapidly as a result of braking vibratory moment - Fig. 9.

a)



b)

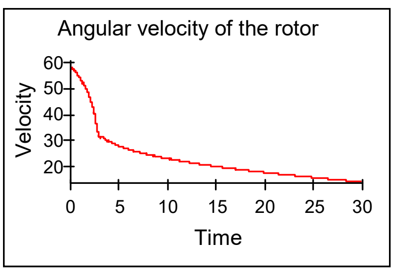
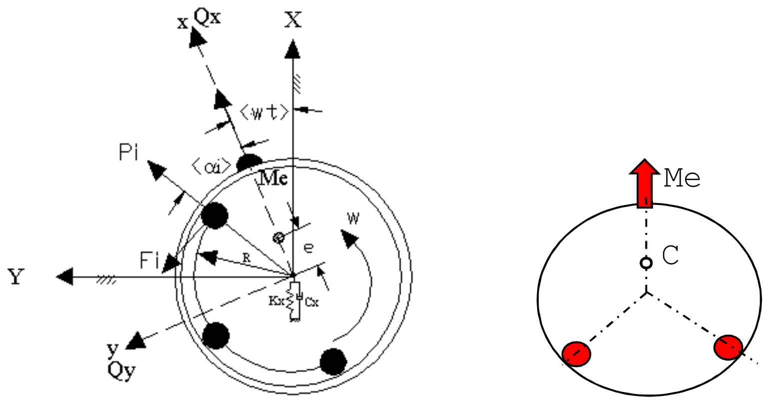


Fig. 9. Behaviour of the rotor without the moment T_m
a) as result of viscous damping
b) as result of vibratory moment

3. Self-balancing of the rotor [Ref. 8 - 10]

The rotor with 1 DOF with a static imbalance *Me* and *N* free-moving balls inside is shown in Fig.3. The rotor is elastically supported in direction x.



a) b)

Fig.10. Unbalanced rotor (a) and the final position of the balls (b)

The vibration of the rotor is defined by

$$M\ddot{x} + c\dot{x} + kx = Me\omega^2 \cos(\omega t - \varphi) + mR\sum_{i=1}^{N}((\omega + \dot{\alpha}_i)^2 \cos(\omega t + \alpha_i) + \ddot{\alpha}\sin(\omega t + \alpha_i)), \qquad (21)$$

and the behavior of the ball is governed by

$$mR\ddot{\alpha}_i = m\ddot{x}\sin(\omega t + \alpha_i) - nR\dot{\alpha}_i = F_{ti} - nR\dot{\alpha}_i \qquad (22)$$

where *M*, *R*, *c*, *k* are the parameters of the rotor, *m*, *R*, *N* are the parameters of the balls.

The solution of Eq. (21) can be approximated as

$$x(t) \cong A_o \cos(\omega t - \varphi) + \sum_{i=1}^{N} A_i \cos(\omega t + \alpha_i - \varphi)\,. \qquad (23)$$

The inertia force acting on the *ith* ball $F_{ti} = m\ddot{x}\sin(\omega t + \alpha_i)$ (24)

From the author's previous works, it is known that the motion depends on the average magnitude of inertia force F_{ti}

$$F_i = \frac{1}{T}\int_{=}^{T} \overline{F}_{ti}(t)\cdot dt\,. \qquad (25)$$

where T is the period of vibration $T=2\pi/\omega$.

By assuming the vibration of the rotor in form eq. (23), the vibration force acting on the *ith* free elements is

$$F_i = -0.5m\omega^2[a_o \sin(\alpha_i + \varphi) + \sum_{j=1}^{N} a_j \sin(\alpha_i - \alpha_j + \varphi)]\,. \qquad (26)$$

The balls change their position with respect to the imbalance under the action of the vibratory force F_i.
When the balls are in a position opposite the imbalance, the rotor vibrations and the vibratory force decrease - Fig.10b. The change of the vibratory force with the change of the position of one ball is shown in Fig. 11. We can see that the ball at $\alpha_f=\pi$ is in a stable equilibrium position for angular velocity of the rotor $\omega>\omega_o$ - $F(\pi)=0$ and $\frac{dF}{d\alpha} < 0$. The balls can compensate for the rotor imbalance only for over-critical speed of the rotor.

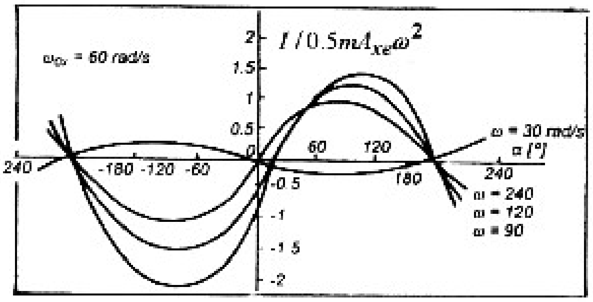


Fig.11. Vibratory force as a function of the ball position
for $\omega > \omega_o$ and $\omega < \omega_o$

When the rotor has two degrees of freedom then the inertial force has a form

$$F_{ti} = m[\ddot{x}\sin(\omega t + \alpha_i) - \ddot{y}\cos(\omega t + \alpha_i)]\,, \tag{27}$$

and the vibratory force can be twice as high as for the rotor with 1DOF. For the rotor with static and dynamic imbalance (Fig.12) the balls should be located on two planes to compensate for these two imbalances. The vibratory force acting on the ball depends on the linear and angular vibrations of the rotor.

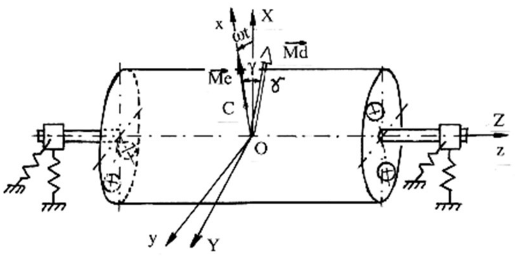


Fig. 12. Balancing in two planes

4. **Synchronous eliminator of vibrations** [Ref. 11 - 12]

The object has a free-moving element (ball or pendulum), which via the action of vibratory forces, can move in synchrony with the excitation and occupy the positions opposite the excitation. The principle of the synchronous eliminator of vibration [11] is very similar to the self-balancing system if the angular velocity ω of the drum with the balls is equal to the excitation frequency.

a)

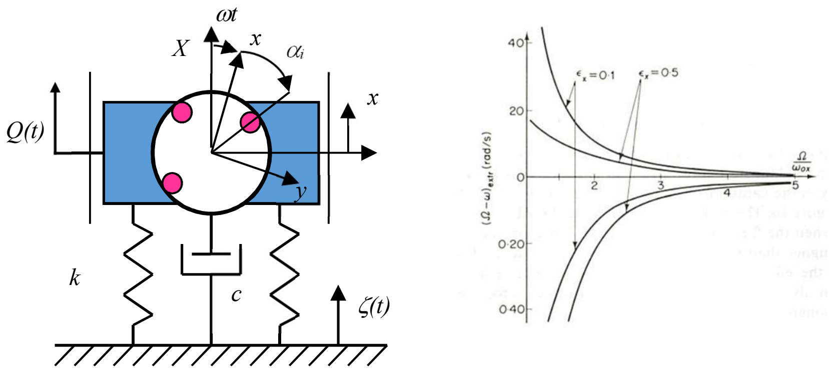


Fig. 13. Synchronous eliminator of vibration (a) and allowable frequency difference (b) $\Delta\omega$ if c=0.05 kg/s.

If $\omega=\Omega$ and there is only viscous resistance to motion, then it is possible to completely eliminate the object's vibrations. When $\omega \neq \Omega$ and small $\Delta\omega=\Omega-\omega$, then some residual vibrations will occur. The maximum allowable difference $\Delta\omega$ depends on the ratio Ω/ω_o as shown in Fig. 13b.

If the object has more degrees of freedom, there should be more free-moving elements on some planes to be able compensate for any excitation; forces and moments [12]. Fig. 14 shows a diagram of the object moving on a plane.

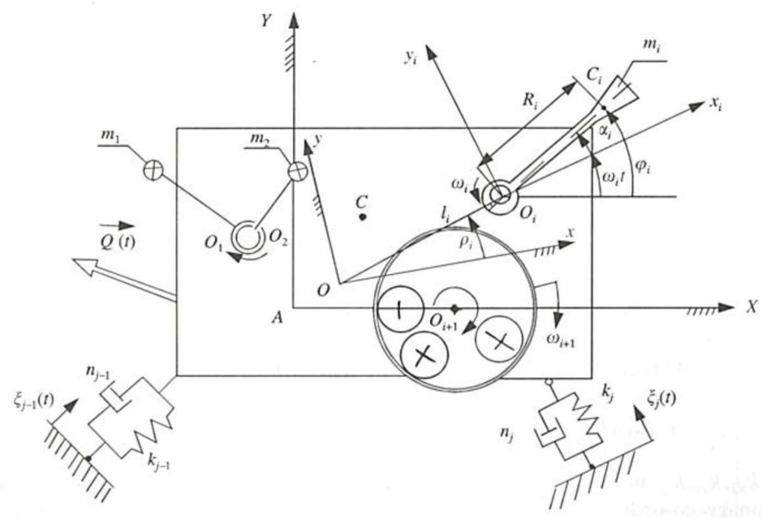


Fig. 14. Object with synchronous eliminators of vibrations

Via the action of the vibratory forces, the free elements organize themselves to compensate for the excitation.

5. Friction with vibration of the base and vibratory transport [Ref. 7, 13]

A block with a mass m is situated on the plane that vibrates in harmonic way in $x_o(t)$, $y_o(t)$ and $z_o(t)$ directions – Fig.15. The motion of the block is the result of force P and three inertial forces:

$$I_x(t) = m\omega^2 a_x \sin\omega t, \quad I_y(t) = m\omega^2 a_y \sin(\omega t + \psi_y), \quad I_z(t) = m\omega^2 a_z \sin(\omega t + \psi_z). \tag{29}$$

where ω is the frequency and a_x, a_y, a_z, ψ_y, ψ_z are the amplitudes and shift angles of the vibrations.
The friction force is defined as

For slipping velocity v=0 and $\sqrt{(P+I_x)^2 + I_y^2} < F_o = \mu N, \quad F_x = P + I_x, \quad F_y = I_y.$ (31)

where the normal reaction is $N = mg - I_y > 0$.

If $v \neq 0$ then $F_x = F_o v_x / v, \quad F_y = F_o v_y / v,$ (32)

where v, v_x, v_x, are the slipping velocity and its components.

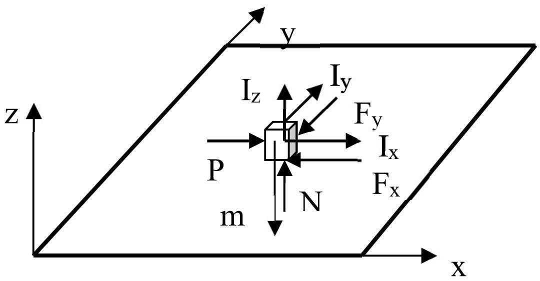


Fig.15. Block on vibrating plane

The behavior of the block on the plane is governed by

$$m\ddot{x} = P + I_x - F_x, \qquad m\ddot{y} = I_y - F_y. \tag{33}$$

If the plane does not vibrate in a vertical direction, then N_o=mg and the minimum force P to move the object is $P_{\min} = \sqrt{N_o^2 - I_y^2} - I_x$.

If the plane also vibrates in a vertical direction, the normal reaction N_o is replaced by the dynamic normal force N.

If the coefficient of friction is determined as $\mu = P_{\min} / N_o$, then it decreases as the amplitude and frequency of the plane vibration increase. The coefficient of friction defined this way is called the *equivalent coefficient of friction*.

Fig. 12 shows how force P_{min} and the equivalent coefficient of friction change with the frequency, and how the object can move on the vibrating plane if $P_{min}<P<N_o$.

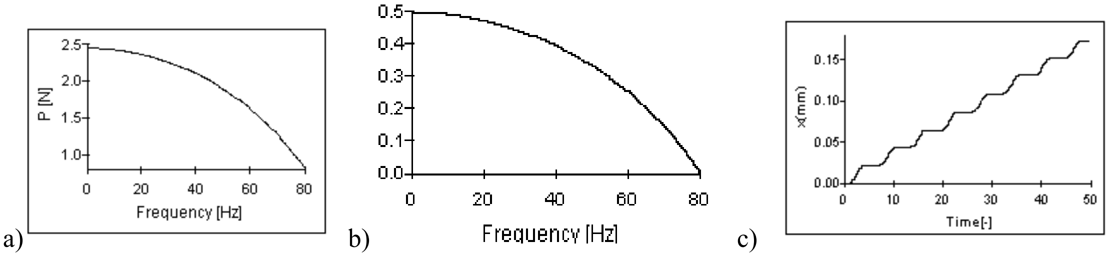


Fig. 15. Minimum force, equivalent coefficient of friction and the motion of the block (50 Hz) for m=0.5 kg, μ_o=0.5, a_x=a_y=10 μm

As shown, the vibratory forces can "decrease" the force needed to move the object. When the equivalent coefficient of friction becomes negative, the object moves without any force P – it is vibratory transport.

6. Conclusions

Via vibratory forces, a system can change its properties, the position of equilibrium may move, and the static position of equilibrium may change to unstable or from unstable to static.
Because of vibratory forces, systems are able to detect the vibration and then organize themselves in such a way to compensate for the excitation and eliminate the vibration. These are the examples of self-organizing systems.
As it was shown, the force necessary to overcome the friction force and displace the object can be much smaller than is normally observed for static friction. The dynamic coefficient of friction (equivalent coefficient of friction) is much smaller than the static coefficient. For high frequency or amplitude, the equivalent coefficient of friction becomes negative, and as a consequence, the motion of the element of the vibrating surface is known as vibratory transport. Controlling the components of vibration of the plane, it is possible to obtain any trajectory of motion and also the locomotion velocity. Locomotion of very small robots is a result of vibrations generated by different actuators.

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