Frequency Response of P-Mode Intrinsic Localized Mode

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Summary. In a coupled oscillator array, nonlinearity and discreteness can cause energy localizations. This type of localization is called an intrinsic localized mode (ILM). Two important types of ILMs are the P-mode ILM, which is an anti-symmetric spatial mode, and the ST-mode, which is a symmetric spatial mode. In this effort, the amplitude of the ILM profile vs. the response frequency is observed for the P-mode ILM.

Introduction

Intrinsic localized modes are nonlinear modes of vibration associated with coupled oscillator arrays. ILMs are large amplitude oscillations, which may cause damage to an array. If properly understood, these energy localizations may be used advantageously in applications; e.g., for sending packets of energy through an array. This localization phenomenon can serve as a paradigm for localization in other systems.
ILMs have been experimentally observed in a range of physical systems, including macro-scale cantilever arrays [1], micro-scale cantilever arrays [2], photonic lattices [3], and Josephson junctions [4]. Analytical and numerical investigations into this phenomenon have also been performed. The restricted normal mode approach has been used to investigate the ST-mode, and it has been discussed that the ST-mode may be considered as a nonlinear vibration mode of an array [5]. More recently, the same approach was used to find the profile for the P-mode ILM [6]. The effects of noise on an ST-mode ILM has been explored both experimentally and numerically in a recent work [7].
Here, the frequency response is investigated for different amplitudes of the P-mode profile. Though this is explored by simulating the unforced and undamped system, this information could elucidate information for the response of the forced P-mode ILM. In the next section, the system of equations for the coupled oscillator array are given, and results of the described method are presented. Concluding remarks are presented in the last section.

Effects of ILM Profile Amplitude to Response Frequency

For the i^{th} oscillator in the coupled oscillator array, the equation of motion is given by

$$\begin{gathered}\ddot{x}_i + c_i\dot{x}_i + \alpha_1 x_i + \beta_1 x_i^3 + \alpha_2(x_i - x_{i+1}) + \alpha_2(x_i - x_{i-1}) \\ +\beta_2(x_i - x_{i+1})^3 + \beta_2(x_i - x_{i-1})^3 = Fcos(\Omega t)\end{gathered} \tag{1}$$

where α_1 is the onsite linear stiffness term, α_2 is the intersite linear coupling term, β_1 is the onsite cubic stiffness term, β_2 is the intersite cubic coupling term, c_i is the damping coefficient that is positive valued, F is the forcing amplitude, and Ω is the forcing frequency. The mass coefficient has been normalized for this equation. Note that all oscillators are identical.
The P-mode ILM profile was calculated with a restricted normal mode approach by the author, in [6]. This approach assumed no external forcing. It was also found that by adding forcing to this system, a beat note is produced in the response. By simulating an array with a reduced size, of just two oscillators, this same phenomenon may be observed. First, the response frequency of the unforced and undamped pair of oscillators is found from a simulation. By forcing the slightly damped pair of oscillators at this frequency, the beat note may be observed in Figure 1.

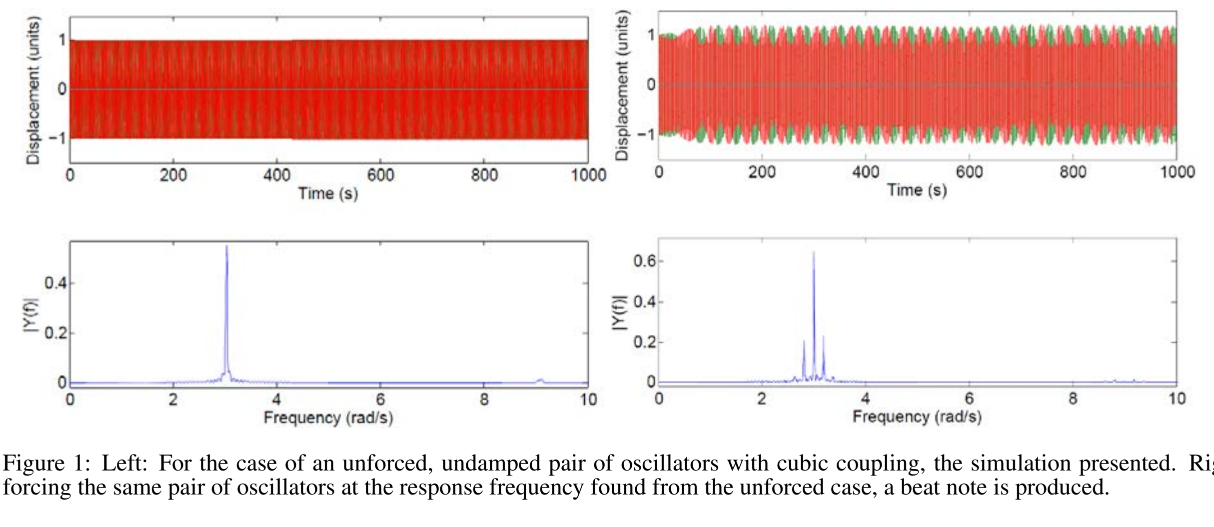

Figure 1: Left: For the case of an unforced, undamped pair of oscillators with cubic coupling, the simulation presented. Right: By forcing the same pair of oscillators at the response frequency found from the unforced case, a beat note is produced.

Usually, the frequency response curve for a nonlinear system is found by quasistatically varying the forcing frequency. Since even a pair of oscillators with cubic coupling may produce this beat note, the full oscillator array of 16 oscillators

was simulated without forcing or damping to avoid the beat note phenomenon. By varying the P-mode ILM profile amplitude, the frequency response of the unforced system was found.
It should be noted that in Figure 2, the four oscillators at the center of the P-mode ILM profile respond with the same frequency, while all of the other oscillators in the array respond with a frequency very close to the linear natural frequency.

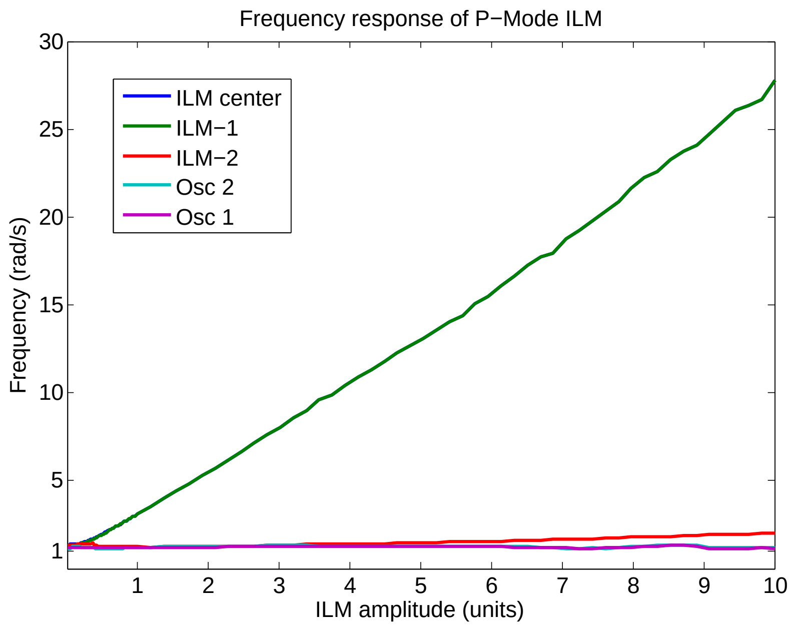

Figure 2: The frequency response vs. P-mode ILM amplitude is shown. By simulating the unforced system from initial conditions producing an ILM, the system oscillates with different frequencies. In the legend, “ILM center” is one of the oscillators at the center of the ILM spatial profile (since the P-mode is anti-symmetric, the two oscillators at the center have the same amplitude and are 180^o out of phase); “ILM-1” is the oscillator to the left of “ILM center”; “ILM-2” is the oscillator to the left of “ILM-1”; “Osc 2” is the oscillator to the right of the left-most oscillator; and “Osc 1” is the left-most oscillator.

Conclusions

Using the P-mode ILM profile found from restricted normal mode analysis, the frequency response for this type of ILM was found by varying the profile amplitude. As can be seen in Figure 2, the 4 oscillators participating in the P-mode ILM localization respond with a very different frequency, when compared to the other oscillators, which are responding with a frequency close to the linear natural frequency. For an ST-mode ILM with relatively little to no cubic coupling, much of the dynamics of the ILM could be predicted by the nonlinear response of a single Duffing oscillator. As a matter of practice, the forced ST-mode is often induced by forcing the oscillator array at a frequency in the region of the single Duffing oscillator’s hysteresis curve, where two stable solutions occur. However, starting from the P-mode ILM profile and forcing the system at the response frequency of the unforced, undamped case, a beat note phenomenon is observed (similar to the response of the oscillator pair presented in Figure 1). Further studies are needed to fully understand this localization phenomenon.

References

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