Extreme Response Mitigation of Stochastically Forced Nonlinear Structures

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Summary. We develop an efficient quantification method for the response statistics of nonlinear multi-degree-of-freedom systems under stocastic, extreme forcing events, emphasizing accurate heavy-tail statistics. This scheme is applied to the design and optimization of small attachments that can mitigate extreme forcing events delivered to a primary system. We then perform design optimization on the nonlinear characteristics of the attachment (assuming a generic, piecewise linear from) and propose a new design that far outperforms optimal cubic energy sink and tuned mass dampers. We note that the proposed quantification and optimization scheme has orders of mangitude smaller computational cost with direct simulation methods such as Monte Carlo.

Introduction

Environmental loads are typically random by nature and are likely to include intermittently occurring extreme events. These determine the system behavior away from the average operating conditions and are important for safety assessment. Examples include high speed crafts in rough seas, wave impacts on offshore platforms, ship capsize events, and structural vibrations due to earthquakes. Accurate reliability assessment for such systems is an important problem because of the possibly catastrophic consequences of a failure to predict or design systems that can safely mitigate the consequences of extreme loading scenarios.

Description of the Method

Consider a three-degree-of-freedom prototype system of high-speed craft, which represents a suspended deck (m_h) attached to a seat (m_s) and a small linear/nonlinear vibration absorber (m_a) (see [1] for details in parameters):

$$\begin{aligned} m_h\ddot{y} &+ \lambda_h\dot{y} + k_h y + \lambda_s(\dot{y}-\dot{x}) + k_s(y-x) + \lambda_a(\dot{y}-\dot{v}) + k_a(y-v) + c_a(y-v)^3 = -m_h\ddot{h}(t) + \sum_{i=1}^{N(t)} \alpha_i\,\delta(t-\tau_i), \qquad (1)\\ m_s\ddot{x} &+ \lambda_s(\dot{x}-\dot{y}) + k_s(x-y) = -m_s\ddot{h}(t),\\ m_a\ddot{v} &+ \lambda_a(\dot{v}-\dot{y}) + k_a(v-y) + c_a(v-y)^3 = -m_a\ddot{h}(t), \end{aligned}$$

where the stochastic forcing (in the form of base excitation) has been decomposed into two parts: a background forcing component $\ddot{h}$ and a rare and large amplitude forcing component in the form of an impulse train. Due to the form of the excitation, the structure of the statistical response is significantly complex, possesing heavy-tails. The idea is to decouple the rare event regime from the background fluctuations to perform response quantification separately and then to synthesize, in order to obtain the full response PDF using a total probability law:

$$f_z(r) = \frac{1-\nu_\alpha\tau^z_{e,\text{dis}}}{\sigma_{z_b}\sqrt{2\pi}}\exp\left(-\frac{r^2}{2\sigma^2_{z_b}}\right) + \nu_\alpha\tau^z_{e,\text{dis}}\int_0^\infty \text{Hist}\{z_{r|\eta}(t\mid n)\}\,f_\eta(n)\,dn, \qquad (2)$$

where z is either the displacement, velocity or acceleration of the seat. Detailed derivations are presented in [1]. This result is a direct extension of the semi-analytical PDF quantification scheme applied to the linear structure in [2].

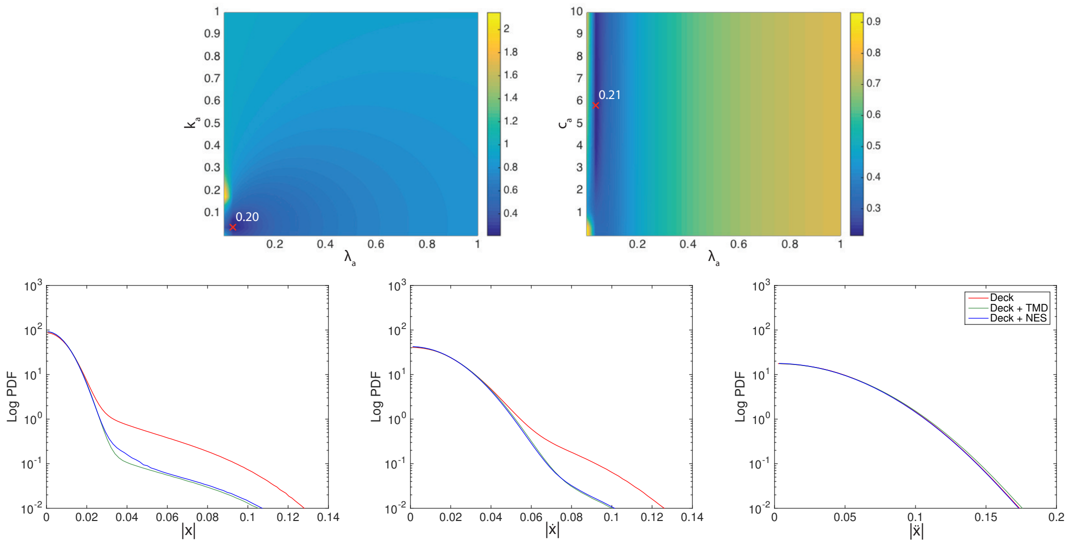

Figure 1: Top: grid search optimization of the suspended deck with tuned mass damper ($c_a = 0$, left) and nonlinear energy sink ($k_a = 0$, right). Bottom: seat response PDF comparisons. No attachment (red), optimal TMD (green), and optimal NES (blue).

Optimization for Extreme Event Mitigation

The developed statistical quantification scheme allows us to explore rare event mitigation properties of different parameters and to perform optimization on the attachment. Such analysis is not practically feasible with a direct Monte-Carlo approach due to the prohibitive computational cost associated with the tail statistics. We adopt the fourth-order moment of the response as a criterion to reflect the severity of extreme events on the primary structure. A small value of fourth-oder moment is desired as it denotes better suppression characteristics of the attachment. The goal is to optimize the performance characteristics of the attachment when its parameters are varied with respect to this severity measure. In Figure 1 we illustrate the results of grid search optimization using the following measure:

$$\gamma = \overline{x_a^4}/\overline{x_o^4}, \tag{3}$$

where x_o is the response without attachment and x_a is the response with attachment. Values less than 1 ($\gamma < 1$) denote effective suppression. Figure 1 shows that the NES is more robust to variations in the attachment parameters over the TMD. This is in line with the fact that NES performs better over a broader spectrum than TMD, which needs to be tuned carefully to the resonance frequency.

Design and Optimization of a piecewise linear NES

Instead of restricting the NES to the canonical cubic design, we perform design optimization over a generic, asymmetric, piece-wise linear family of NES. This NES has a small amplitude region with identical characteristics with the TMD and a larger amplitude region associated with positive (negative) displacements, characterized by slope a_1 (a_{-1}). The optimization is performed using the following measure:

$$\gamma' = \overline{x_n^4}/\overline{x_t^4}, \tag{4}$$

where x_t is the response with the optimal TMD (from the previous optimization result in Figure 1) and x_n is the response with the proposed NES. The result of the design optimization and the corresponding optimal PDFs are shown in Figure 2. We emphasize the asymmetric character of the derived optimal NES as well as its significantly improved performance compared with the TMD and cubic NES.

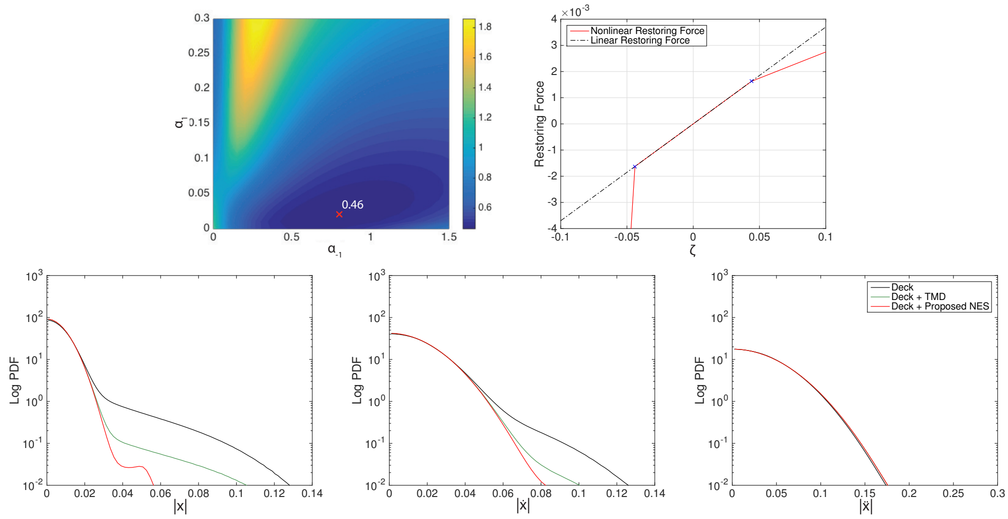

Figure 2: Top: grid search optimization with respect to α_{-1} and α_1 (left) and the corresponding optimal restoring curve (right). Bottom: seat response PDF comparisons. No attachment (black), optimal TMD (green), and optimal proposed NES (red). See [1] for details.

Conclusions

The key contribution of this work is the design and optimization of a NES that can optimally mitigate and suppress the stochastic extreme forcing events delivered to the primary system. We examined the suppression of extreme responses via TMD and cubic NES through parametric optimization by minimization of the forth-order response moments. We then proposed an asymmetric NES that far outperforms the optimal TMD and cubic NES for the considered problem.

References

[1] H. K. Joo, M. A. Mohamad, and T. P. Sapsis. “Extreme events and their optimal mitigation in nonlinear structural systems excited by stochastic loads”. In: *Submitted* (2017).

[2] H. K. Joo, M. A. Mohamad, and T. P. Sapsis. “Reliability of structural systems subjected to extreme forcing events”. In: *Submitted* (2016).