Nonholonomic lane change maneuvers for connected and autonomous vehicles

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Summary. A nonholonomic model is proposed to describe the longitudinal, lateral and yaw dynamics of automobiles. The equation of motion is derived using the Appellian approach while assuming rigid wheels. Controllers that regulate the steering and the powertrain simultaneously are developed to allow the autonomous vehicle to execute a large variety of maneuvers. In particular, by assigning the steering angle and the driving force, lane change maneuvers are executed in an open loop fashion. Moreover, nonlinear feedback controllers are designed to allow lane changes when the autonomous vehicle monitors the motion of other vehicles using sensors and wireless vehicle-to-vehicle (V2V) communication.

Introduction

Autonomous vehicle technologies are making their way to public roads through the efforts of traditional and nontraditional automakers. These vehicles are able to perceive their environment through a variety of sensors (radars, cameras, lidars), reason and make decisions using on-board computers, and actuate themselves by controlling the steering and the powertrain. Moreover, wireless vehicle-to-vehicle (V2V) communication may allow them to obtain kinematic information from surrounding vehicles, including information that is very difficult (if not impossible) to extract from sensory data (like lateral acceleration and yaw rate of other vehicles). When using V2V communication even the motion of vehicles beyond the line of sight can be monitored. These technologies demand for models that can describe the motion of the vehicle with high fidelity while being simplistic to remain feasible for control design. Moreover, the applied controllers also need to be kept below a certain complexity in order to make them implementable on board. We target these problems in this paper.

Appell equations

We consider the Appellian approach [1, 2] to derive the equation of motion as this allows us to handle nonholonomic rolling constraints. We consider the mechanical model of a front-wheel steer, rear-wheel drive vehicle shown in Fig. 1(a). The front wheels are represented as a single massless wheel and the same is done for the rear wheels which results in a so-called bicycle model. The mass of the car is denoted by m and its moment of inertia about the center of mass is denoted by $J_{\rm G}$. The wheelbase is L and the distance between the center of mass and the rear axle is d. The steering angle is denoted by γ while the driving force is denoted by F. To describe the location and orientation of the vehicle we use the coordinates of the center of mass (x, y) and the yaw angle ψ.
The wheels are not allowed to slip laterally and the corresponding kinematic constraints can be formulated as

$$\begin{aligned} \dot{x}\sin\psi - \dot{y}\cos\psi + \dot{\psi}d &= 0\,, \\ \dot{x}\sin(\psi+\gamma) - \dot{y}\cos(\psi+\gamma) - \dot{\psi}(L-d)\cos\gamma &= 0\,, \end{aligned} \tag{1}$$

for the rear and the front wheels, respectively. As this is a three degree-of-freedom system with two kinematic constraints, one needs to choose $3 - 2 = 1$ pseudo velocity to unambiguously describe the motion. Here we choose the longitudinal velocity of the vehicle

$$u = \dot{x}\cos\psi + \dot{y}\sin\psi\,. \tag{2}$$

Solving (1,2) for $\dot{x}, \dot{y}, \dot{\psi}$, we obtain

$$\begin{aligned} \dot{x} &= u\big(\cos\psi - \tfrac{d}{L}\sin\psi\tan\gamma\big)\,, \\ \dot{y} &= u\big(\sin\psi + \tfrac{d}{L}\cos\psi\tan\gamma\big)\,, \\ \dot{\psi} &= u\tfrac{1}{L}\tan\gamma\,. \end{aligned} \tag{3}$$

The acceleration energy can be expressed as

$$S = \tfrac{1}{2}m\big(\ddot{x}^2 + \ddot{y}^2\big) + \tfrac{1}{2}J_{\rm G}\ddot{\psi}^2 = \tfrac{1}{2}(m + m_0\tan^2\gamma)\dot{u}^2 + m_0\tfrac{\tan\gamma}{\cos^2\gamma}\dot{\gamma}\,u\,\dot{u} + (\cdots\text{terms without } \dot{u}\cdots)\,, \tag{4}$$

where the time derivative of (3) was substituted and $m_0 = (m\,d^2 + J_{\rm G})/L^2$. By calculating the virtual power of the active force F, that is, $\delta P = F\delta u$, we obtain the pseudo force $\Gamma = F$. Thus, the Appell equation yields

$$\frac{\partial S}{\partial \dot{u}} = \Gamma \quad \Rightarrow \quad \dot{u} = \frac{F - m_0\frac{\tan\gamma}{\cos^2\gamma}\dot{\gamma}\,u}{m + m_0\tan^2\gamma}\,. \tag{5}$$

The differential equations (3,5) describe the combined lateral, longitudinal, and yaw dynamics of the automobile.

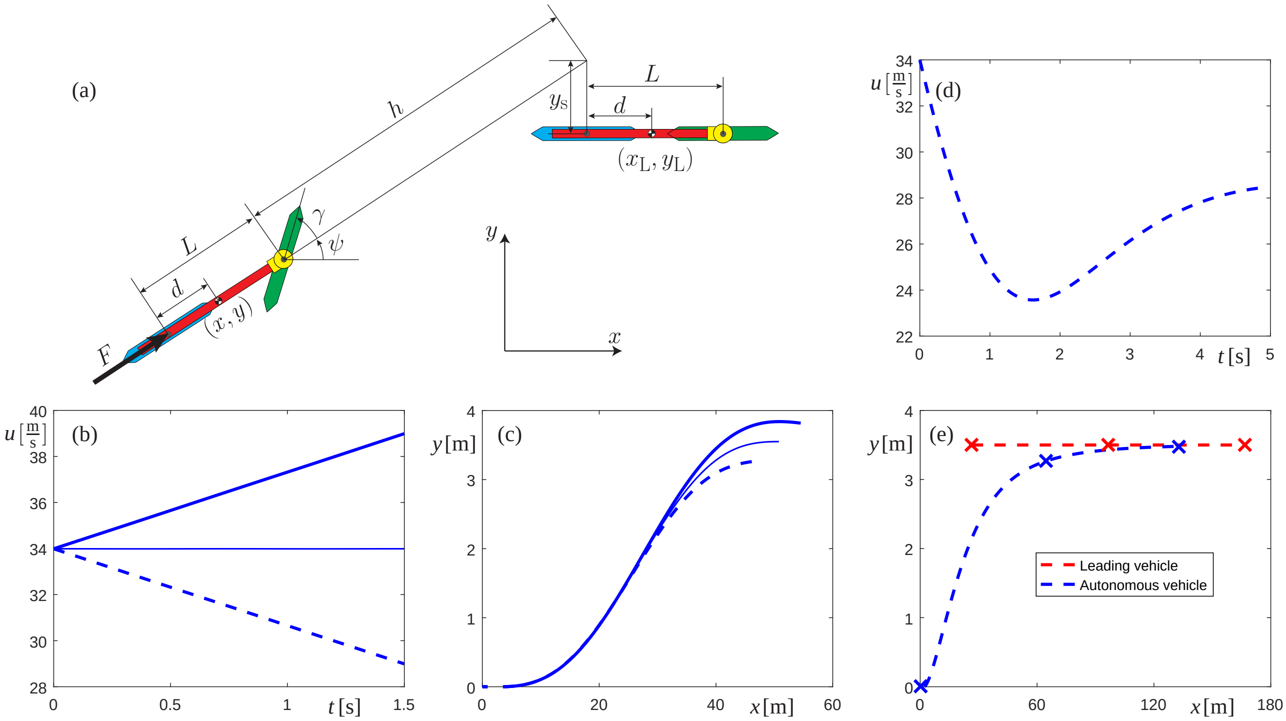

Figure 1: (a) Mechanical model used in the paper. (b,c) Lane change with open loop control: the longitudinal velocity u is shown as a function of time t and the trajectory is plotted in the (x, y)-plane for one period of the steering input (6) with amplitude $A = 0.0215$ [rad] and frequency $\omega = 4\pi/3$ [rad/s] while applying constant driving forces $F = 0$ [N] (thin solid curve), $F = 5000$ [N] (thick solid curve), and $F = -5000$ [N] (thick dashed curve). (d,e) Lane change with feedback control utilizing the kinematic information of another vehicle in the neighboring lane: the longitudinal velocity and the trajectory are shown for the feedback law (7) with gains $p = 0.5$ and $q = 1500$ [kg/s]. The vehicles' positions at 0, 2.5, and 5 [s] are marked by $\times$-s. The parameters used are $m = 1500$ [kg], $L = 2.5$ [m], $d = 1.25$ [m], $J_G = 2500$ [kg m^2].

Control design for lane change maneuvers

In order to control the motion of the autonomous vehicle one shall assign the steering angle γ and the driving force F as a function of time. First, we execute a lane change using an open loop control strategy. In particular, we assign the steering angle as

$$\gamma(t) = A \sin(\omega t)\,, \tag{6}$$

while applying a constant driving force F. The corresponding results are shown in Fig. 1(b,c). One may observe on panel (b) that the longitudinal velocity remains approximately constant when the driving force is zero, increases approximately linearly when the force is positive and decreases approximately linearly when the force is negative. The corresponding trajectories differ in the three different cases as shown in panel (c); see the caption of the figure for the parameter values. Such open loop controllers may be realized by simply relying on GPS information and a digital map, which is feasible on an open road with no other vehicles around.

When the autonomous vehicle is surrounded by other vehicles, lane change maneuvers need to be executed while responding to the motion of those vehicles that can be monitored by sensors and/or by V2V communication. Here we consider a lane change scenario when the autonomous vehicle moves into a lane with a slower vehicle ahead. In order to respond to the motion of the leading vehicle the autonomous vehicle utilizes the distance headway h and the lateral distance y_s that can be obtained by cameras or calculated using the GPS coordinates (x, y) and yaw angle ψ of the autonomous vehicle and the GPS coordinates (x_L, y_L) of the leading vehicle monitored through V2V communication; see Fig. 1(a). Then we propose the nonlinear controller

$$\gamma = -p\, y_s/h\,, \qquad F = q(V(h) - u)\,, \tag{7}$$

where p and q denote the control gains while the monotonously increasing function V determines the desired velocity as a function of the distance headway; see [3] for details. The results in Fig. 1(d,e) show that the autonomous vehicle executes a successful lane change while keeping a safe distance from the vehicle ahead.

References

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