Analysis of dry galloping on inclined cables under stationary wind

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Summary. The critical conditions and post-critical behavior of stay-cables in case of dry galloping due to stationary wind are analyzed; cases of different inclinations of the chord are taken into account. After evaluating the initial configuration under the effect of self-weight and static pre-stress, the nonlinear exact equations of motion are written for elastic flexible cables, including the contributions of aeroelastic forces. Wind effects on the yawed cylinder are considered in a quasi-steady framework and can induce dry galloping, i.e. galloping when neither rain rivulet nor ice accretion is present on the cross-section of the cable, which is actually circular. A Galerkin projection of the partial differential equations and the analysis of the discrete system of equations of motion, using both numerical and perturbation techniques, allow to determine the instability threshold and the post-critical evolution of the cable.

Introduction

Stay-cables are structural members typically used in important engineering structures like bridges and towers, and where the low weight-to-stiffness ratio is a crucial asset. In [1] the free dynamics of low-inclined stay-cables was evaluated rotating the reference system to the chord and using a suitable modification of the equations valid for horizontally supported cables. In [2] a specific model for inclined cables was introduced, and natural frequencies and modal shapes were asymptotically obtained, showing hybridization of the modes and loss of crossover of the frequencies, phenomena not captured by adaptation of the equations valid for horizontally supported cables. Nonlinear models have been introduced lately to study forced dynamics. In [3] cable-structure models as an adaptation of horizontal cables were introduced to study vibrations and to design actuators, prefiguring the use of numerical techniques in the derivation of the solution. In [4], numerical and analytical techniques were used to study the nonlinear free dynamics of inclined cables, discussing the effect of the static condensation of the longitudinal displacement. In [7, 8, 9], a consistent model of beam-like cable was introduced to take into account the effect of the twist and bending in horizontally supported cables; examples on galloping of iced cross-sections are carried out. In [5], the combined effect of harmonic base motion and galloping due to the presence of a rain rivulet on the cross section of a stay-cable was considered, showing possible interaction among the different sources of excitation. In [6], a complete nonlinear model of flexible inclined cables was introduced and then, after evaluation of the order of magnitude of some terms, simplifications to the model were suggested. In the last decade (e.g., [10] the mechanism inducing dry galloping of yawed circular cylinders has been extensively analyzed by means of wind tunnel tests. Despite unsteadiness and spatial variation of the flow can be significant during the cable motion [11], an approximate description through a quasi-steady model seems possible (e.g., [11, 12]). In this paper, starting from the nonlinear model proposed in [6], the analysis of galloping-like instability in yawed conditions is performed for significantly inclined long stays with different inclination of the chord. Natural frequencies and normal modes are numerically obtained, taking into account the possible significant variation along the span of both the curvature and the pre-stress already in the static configuration, and highlighting hybridization of both in-plane and out-of-plane modal shapes. Then, after a Galerkin projection of the partial differential equations on the first significant modes, the discretized system of equations of motion is addressed by means of both numerical and perturbation techniques, in order to get information on the stability conditions where dry galloping is triggered as well as to evaluate the post-critical dynamics.

Equations of motion

The static configuration

As a first step, the static configuration of the pre-stressed stay-cable is obtained for inextensible material. Following the guidelines of [5] and applying the equilibrium equations when self-weight is considered, the static configuration is parametrically described in terms of horizontal $\bar{x}$ and vertical $\bar{y}$ components of the position of the axis line of the cable at abscissa s:

$$\bar{x}(s) = \frac{H}{mg}\Big[\arcsin\Big(\frac{mgs}{H}+c\Big) - \arcsin(c)\Big], \qquad \bar{y}(s) = \frac{H}{mg}\Big[\sqrt{1+\Big(\frac{mgs}{H}+c\Big)^2} - \sqrt{1+c^2}\Big] \tag{1}$$

where H is the horizontal component of the pre-stress, m the mass linear density, g the gravity acceleration and c a constant. Imposing the boundary conditions $\bar{x}(l) = x_l$ and $\bar{y}(l) = y_l$, i.e. the position of the right support, located at abscissa $s = l$ which is the length of the cable, one can numerically evaluate e.g. H and c, and fully know the initial configuration by means of Eqs. (1). Then, evaluation of the rotation $\bar{\vartheta}$ of the tangent to the axis line is easily obtained: $\tan(\bar{\vartheta}(s)) = \frac{mgs}{H} + c$, and consequently the curvature of the cable is $\bar{\kappa}(s) = \bar{\vartheta}'(s)$ and the cable pre-stress is $\bar{T}(s) = \frac{H}{\cos(\bar{\vartheta}(s))}$, where prime stands for differentiation with respect to s.

The incremental equations of motion

Once incremental load vector $\tilde{\mathbf{b}}$ (including aeroelastic forces) and nonlinear strain of the elastic cable, namely $\varepsilon(s) = \sqrt{(1+u'+\bar{k}v)^2+(v'+\bar{k}u)^2+w'^2} - 1$ are considered, where $u(s), v(s), w(s)$ are the components of the dynamic

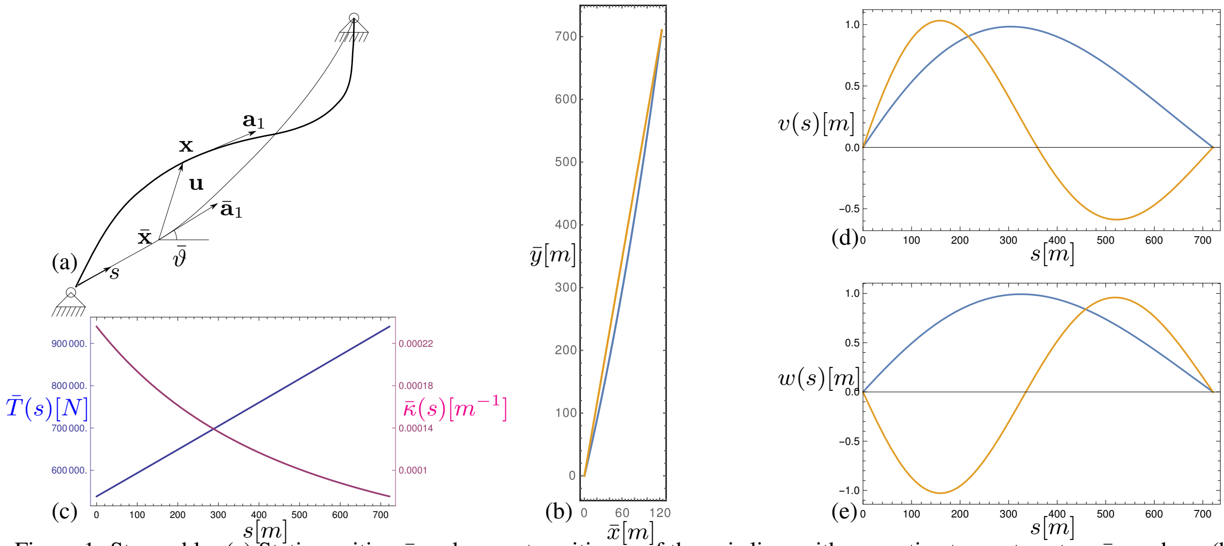

Figure 1: Stay-cable. (a) Static position $\bar{\mathbf{x}}$ and current position $\mathbf{x}$ of the axis line, with respective tangent vectors $\bar{\mathbf{a}}_1$ and $\mathbf{a}_1$; (b) Static configuration for the example (in blue), with inclined chord (in orange); (c) static pre-stress $\bar{T}$ (in blue), and curvature $\bar{\kappa}$ (in magenta) as function of s; (d) in-plane modes: first (blue) and second (orange); (b) out-of-plane modes: first (blue) and second (orange).

displacement $\mathbf{u}(s)$ of the axis line along its tangent, normal and binormal directions indicated as $\bar{\mathbf{a}}_1, \bar{\mathbf{a}}_2, \bar{\mathbf{a}}_3$, respectively (see Figure 1a), the vector form of the equations of motion turns out to be

$$[(\bar{T} + EA\varepsilon)\mathbf{a}_1]' - \bar{T}'\bar{\mathbf{a}}_1 - \bar{T}\bar{\kappa}\bar{\mathbf{a}}_2 + \tilde{\mathbf{b}} = m\ddot{\mathbf{u}}(s) \tag{2}$$

where EA is the axial stiffness of the cable, $\mathbf{a}_1$ is the (unknown) tangent vector to the current configuration of the axis line, whose expression is $\mathbf{a}_1 = \frac{1}{1+\varepsilon}(\bar{\mathbf{a}}_1 + \mathbf{u}')$ and the dot stands for time differentiation. As a preliminary result, the static configuration of a cable of length $l = 700$ m, inclination of the chord of 80^o and mechanical parameters coherent with a real stay are presented in Figure 1b. The corresponding static pre-stress $\bar{T}$ and curvature $\bar{\kappa}$ are shown as function of s in Figure 1c, showing a significant variation along the span. First two in-plane modes and out-of-plane modes are shown in Figure 1d-e, highlighting hybrid shapes. Further results, descending from the Galerkin projection of Eq. (2) on the four modes shown, and related to critical and post-critical analysis of the cable in conditions of dry galloping are currently under investigation.

Conclusions

The dynamic behavior of inclined yawed stays is analyzed using a complete model able to describe in-plane and out-of-plane hybrid modes. The static configuration is evaluated, descending from equilibrium conditions when self-weight is applied, and then nonlinear equations of motion considering the aeroelastic forces under a quasi-steady framework are written. A Galerkin procedure projecting the partial continuous system on the hybrid modes leads to a discrete system of ordinary differential equations, able to provide critical conditions of galloping-like instability and to describe post-critical evolution of the inclined cable.

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