Optimization Criterions of a Multi-Time-Delay Controlled Isolation System with Asymmetrical Nonlinearity

Xiuting Sun [*], Shu Zhang [**], Jian Xu [**], Huijie Yu [*], Shenglong Wang [*], Yao Yan [***]
[*] *School of Mechanical Engineering, University of Shanghai for Science and Technology, Shanghai, P.R. China*
[**] *School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai, P.R. China*
[***] *School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu, P.R. China*

Summary. Based on the analysis of multiple nonlinear vibration properties, optimization criterions for different types of excitations and structural nonlinearities are proposed for an isolation platform with loading. In order to improve the isolation effectiveness in a wide frequency band, feedback control considering inherent and adjustable time delays is introduced into this system. The main results show that without changing the High-Static-Low-Frequency advantage brought by the nonlinear isolation structure, the isolation effectiveness could be optimized in a wide frequency band for the optimization of structure and time-delayed control.

The nonlinear isolation system with time-delayed feedback control

Fig. 1 is the model of a time-delayed control nonlinear vibration isolation system. Normally, the negative-stiffness component could be realized by three structures, which are per-compression springs [1-2], buckling beams [3] and pre-compression Scissor-Like Structures (SLSs) [4]. In this paper, considering the stability of springs and the potential application in multi-direction isolation, the SLSs are utilized for the realization of nonlinear stiffness property as Fig. 1 (b).

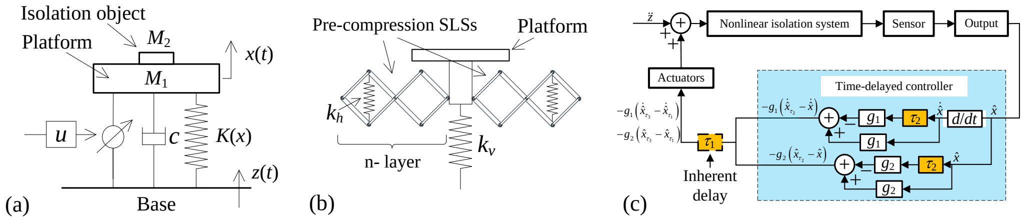

Fig. 1 Model of a controlled isolation platform with loading M_2 and time-delayed active control.

We assume that the control signal contains both the displacement and velocity information as shown in Fig. 1 (c). The inherent time delay τ_1 is considered for the signal computation and transformation in the control loop, and time delay τ_2 is introduced artificially into the control as an adjustable control parameter, thus the control is written as

$$u(\boldsymbol{\mu},\boldsymbol{\tau})=-g_1\left[\dot{\hat{x}}(t-\tau_1-\tau_2)-\dot{\hat{x}}(t-\tau_1)\right]-g_2\left[\hat{x}(t-\tau_1-\tau_2)-\hat{x}(t-\tau_1)\right]. \tag{1}$$

When the adjustable time delay is as τ_2=0, the feedback control has no effect on the dynamics since u=0, and thus the effect of the time delay τ_2 and its control mechanism could be highlighted. By Lagrange principle, the dynamical model of the isolation platform with multi-time-delayed control is as

$$(M_1+M_2)\ddot{\hat{x}}+K(\hat{x})+c\dot{\hat{x}}=-(M_1+M_2)\ddot{z}-g_1\left(\dot{\hat{x}}_{\tau_3}-\dot{\hat{x}}_{\tau_1}\right)-g_2\left(\hat{x}_{\tau_3}-\hat{x}_{\tau_1}\right)-M_2 g, \tag{2}$$

where $K(\hat{x})$ is the nonlinear stiffness function. Because the modeling is around the zero equilibrium, utilizing the Taylor series expansion at $\hat{x}=0$, it can be obtained that

$$\tilde{K}(\hat{x})=\left(-k_v+2k_h l_{s2}/n^2 l_2\right)\Delta_2+\left(k_v\hat{x}-2k_h l_{s2} l_1^2/n^2 l_2^4\right)\hat{x}-3k_h l_0 l_1^2\hat{x}^2/n^4 l_2^6\,\Delta_2+k_h l_0 l_1^2\left(n^2 l_1^2+\Delta_2^2\right)\hat{x}^3/n^6 l_2^8. \tag{3}$$

Dynamic model and solutions of steady states

The expectation of time-delayed control is not only to broaden the isolation effective band from ultra-low frequency but also eliminate the bifurcation and instability. Utilizing the method of multiple scales, the normal form of the solution is as

$$\begin{cases}\dfrac{da}{dT_2}=\Phi(a)-\dfrac{z_0\Omega\sin\varphi}{2},\\ a\dfrac{d\varphi}{dT_2}=\Psi(a)-\dfrac{z_0\Omega\cos\varphi}{2}.\end{cases} \tag{4}$$

In order to evaluate the optimization effect of control parameters on isolation effectiveness, the vibration displacement transmissibility T_d is defined as

$$T_d=\|x\|/\|z\|=\left\|a\cos(\Omega\tilde{t}+\varphi)+\tilde{z}_0\cos\Omega\tilde{t}\right\|/\left\|\tilde{z}_0\cos\Omega\tilde{t}\right\|=\sqrt{1+2a\cos\varphi/\tilde{z}_0+(a/\tilde{z}_0)^2}. \tag{5}$$

where $\tilde{z}_0$ is the amplitude of excitation, a the amplitude of relative motion and $\cos\varphi$ the phase.

Nonlinear vibration properties and optimization of control parameters

Optimization criterion for impact excitation

In order to damping the free vibration for impact excitation quickly, the optimization criterion for impact excitation is proposed as

$$I_1\left(\tau_{k_2}\right)=\min\left\{d\Phi/da\right\}. \tag{6}$$

The optimization criterion could choose the optimum time delay τ_{k2} for fastest dissipation of free vibration. Fig. 2 shows the optimal values of time delay τ_{k2} and the comparison of free vibration for the optimal parameter and other values for different control coefficient $\tilde{g}_2$ as increasing the inherent time delay τ_{k1}.

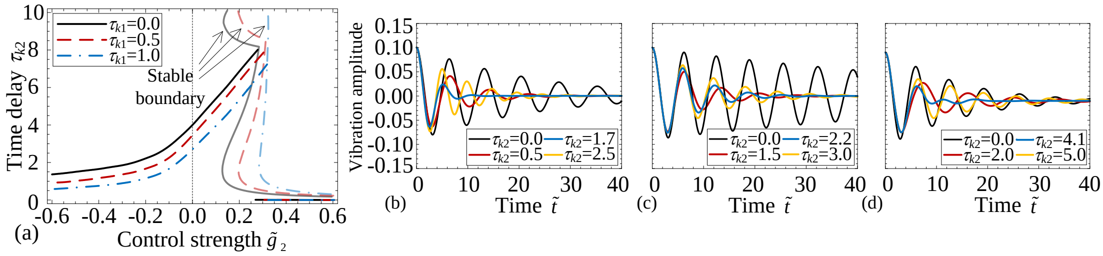

Fig. 2 Optimum values of time delay τ_{k2} for impact excitation and the relevant time series of free vibration.

The vibration response for the optimum time delay and other values are compared in Figs. 2 (b)-(d) and the vibration could be damped fastest for the optimum time delay. Therefore, the vibration time series verify the optimization criterion for impact excitation proposed as Eq. (6).

Optimization criterion for harmonic excitation

For hard-spring nonlinearity, the vibration amplitude could jump to a large value at Ω_1, from where it regards that the isolation is effective, while the frequency band for multi-steady states also begins from Ω_1. Therefore, for hard-spring property, based on the definitions of effective isolation frequency band as $[\Omega_1, \infty)$ and the multi-steady states band as $[\Omega_1, \Omega_2]$, the optimization criterion is proposed as

$$I_2\left(\tau_{k2}\right)=\min\left\{\Omega_1\right\}\&I_3\left(\tau_{k2}\right)=\min\left\{\left|\Omega_2-\Omega_1\right|\right\}. \tag{7}$$

Fig. 3 shows the optimum time delay for different displacement feedback control gain coefficient $\tilde{g}_2$ and the comparison of amplitude-frequency curves among the optimum time delay and other values.

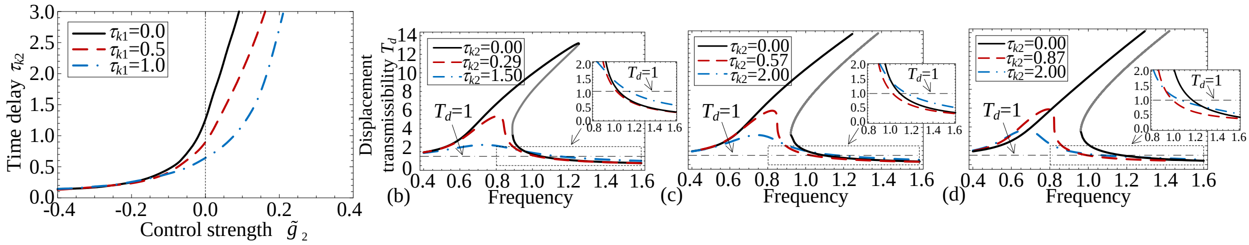

Fig. 3 Optimal values of adjustable time delay τ_{k2} and the relevant amplitude-frequency curves.

From Fig. 3, it reveals that for choosing the optimum time delay (corresponding to the amplitude-frequency curve in Red Dashed Lines), the frequency at jumping-up point is smallest, and thus the effective isolation frequency band is largest. Also, the optimum time delay reduces the multi-steady states band to zero. The comparison demonstrates the effective isolation frequency band, multi-steady states phenomenon and isolation effectiveness in high frequency band are optimized simultaneously.

Conclusions

This paper provides the design, optimization and application of nonlinear isolation system with feedback control including adjustable time delay and inherent time delay. Based on multiple nonlinear vibration properties, the optimization criterions are proposed and the appropriate values of adjustable control parameters for different types of excitations and structural nonlinearities are determined. The optimal values of control parameters could optimize the isolation effectiveness and satisfy multiple requirements without changing the structural nonlinearity in practices.

References

[1] Abolfathi, A., Brennan, M.J., Waters, T.P., Tang, B. (2015) On the effects of mistuning a force-excited system containing a quasi-zero-stiffness vibration isolator. Journal of Vibration and Acoustics **137(4)**.

[2] Hao, Z., Cao, Q. (2014) The isolation characteristics of an archetypal dynamical model with stable-quasi-zero-stiffness. Journal of Sound and Vibration **340:** 61–79.

[3] Huang, X., Liu, X., Sun, J., Zhang, Z., Hua, H. (2014) Vibration isolation characteristics of a nonlinear isolator using Euler buckled beam as negative stiffness corrector: a theoretical and experimental study. Journal of Sound and Vibration **333(4)**: 1132-1148.

[4] Sun, X., Jing, X. (2015) Multi-direction vibration isolation with quasi-zero stiffness by employing geometrical nonlinearity. Mechanical Systems and Signal Processing **62**: 149-163.