# Numerical Method for Nonlinear Vibration of Contact Joint Structures

Loïc Salles\*, Luca Pesaresi\* and Jason Armand \*

\*Vibration University Technology Center, Department of Mechanical Engineering, Imperial College,

London, UK

<u>Summary</u>. This work focusses on modelling of structured with localized non-linearities. The scales involved in vibration of joint structures are very different: macroscale (mm) for the vibration to microscale at the asperity level of the contact. Different approaches have been developed in Vibration University Technology Centre. This paper will summarise the strategies that have been developed and implemented in the in-house code FORSE. Numerical examples will illustrate the proposed methods with focus on bolted joint beam system where the motion at the frictional interfaces can lead to a highly non-linear dynamic response and cause fretting wear at the contact. The latter changes the contact conditions of the interface and consequently impact the non-linear dynamic response of the entire assembly. The paper will be concluded by a discussion on the future strategies concerning numerical tool and experimental validations in order to improve accuracy of vibration analysis with contact joints.

## Introduction

The assembly of single components into a more complex structure always leads to the presence of a joint. Contact joints are the most common way to assemble components, which introduces non-linearities in the system. An accurate prediction of vibration for those structures is still a challenge. A method based on harmonic balance method is proposed in this study to capture the non-linear behaviour of the joint. The method permit to calculate the nonlinear frequency response of the structure. The contact surfaces are discretized using zero-thickness finite element [1] with different contact law to take into the effect of the roughness of the contact surface. The proposed approach is illustrated on a straight beam (Brake-Reuss beam) with a simple lap joint with three bolts. This system has been developed and experimentally analysed by Sandia, Stuttgart University and Imperial College London in order to provide a benchmark for nonlinear vibration methods [5].

#### Method

The approach used for the numerical analysis is based on an existing multi-harmonic balance (MHBM) solver coupled with a model reduction technique, discussed in detail in [2, 3]. This flexible methodology can be applied to predict the non-linear dynamic response of large FE models which contain non-linear joints and are subject to harmonic excitation. Due to the friction forces arising at the contact interface, the equation of motion for a contact joint system are non linear, and it can be written in the following form:

$$M\ddot{\boldsymbol{u}} + C\dot{\boldsymbol{u}} + K\boldsymbol{u} = \boldsymbol{F}_{ex}(t) - \boldsymbol{F}_{NL}(\boldsymbol{u}, \dot{\boldsymbol{u}}), \tag{1}$$

where M,C and K are the matrices of mass, damping and stiffness respectively;  $F_{ex}(t)$  is the periodic external excitation and  $F_{NL}(u, \dot{u})$  the nonlinear contact forces.

The response for each DOF of the system is expressed as a Fourier series truncated at the  $n^{th}$  harmonic:

$$u(t) = Q_0 + \sum_{j=1}^{nh} Q_c^j \cos(j\omega t) + Q_s^j \sin(j\omega t)$$
<sup>(2)</sup>

A Galerkin procedure is performed to project the equation of motion (1) which lead to the following non-linear algebraic system:

$$\boldsymbol{Z}(\omega)\tilde{\boldsymbol{Q}} = \tilde{\boldsymbol{F}} - \tilde{\boldsymbol{F}}_{nl} \tag{3}$$

The contact stress are defined using a compliance law for the normal pressure  $p_N$  and Jenkins element for the tangential stress  $\sigma_T$ .

$$p_N = \alpha \left\langle \delta_N^{beta} \right\rangle_+ \quad \sigma_T = k_T(\delta_N) \delta_T \tag{4}$$

where  $\delta_N$  is the distance between the two contact surfaces,  $\delta_T$  is the tangential elastic displacement;  $\alpha$  and  $\beta$  are the coefficient of the normal contact law, they depend on the contact roughness;  $k_T$  is the tangential contact stiffness and it is defined by  $\frac{2(1-\nu)}{2-\nu}\alpha\beta\delta^{\beta-1}$  with  $\nu$  the Poisson ratio. This approach is original compared to current practice where contact stiffness are assumed constant [3]. The contact forces are expressed in the nodes of the finite element mesh at the contact surface. They are calculated using zero-thickness finite element and Gaussian quadrature. The Fourier coefficient of the contact stress are calculated at each Gaussian points using an alternating frequency time procedure [4].

## **Numerical Example**

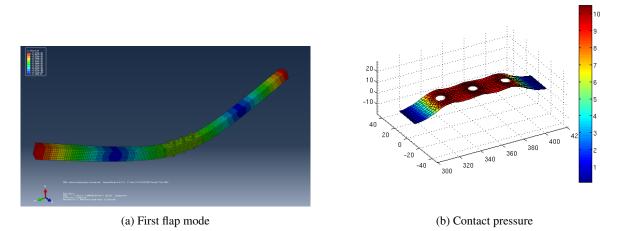


Figure 1: First flap mode of the bolted beam and contact pressure in the joint area

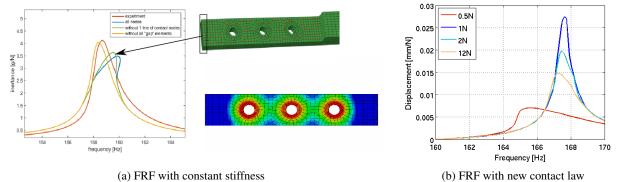


Figure 2: Frequency response with proposed methods

The first flap mode of the studied structures is shown in Fig 1a. The profile of the pressure in the contact area obtained with the proposed contact law is illustrated in Fig. 1b. The Frequency response obtained with the proposed contact law and the one with constant contact stiffness are shown on Fig. 2. It can be seen from those figures that the choice of contact change the behaviour of the frequency response. The proposed approach allows to get a response closer to the one experimentally observed.

### References

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