Solitary Waves in Dimer Binary Collision Model: A Comparative Study with Granular Dimers

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Summary. Solitary wave propagation in nonlinear diatomic (dimer) lattice is a very interesting aspect of research in the study of nonlinear lattices. Such waves have been recently found to be supported by essentially nonlinear granular dimer lattices at a discrete spectrum of mass ratio (the only system parameter governing the dynamics). In this work we explore solitary wave propagation in dimer binary collision (BC) model. Interestingly, the dimer BC model supports solitary wave propagation at a discrete spectrum of mass ratios similar to that observed in granular dimers. Further, we report a qualitative and one-to-one correspondence between the spectrum of mass ratio supporting solitary waves in dimer BC model and granular dimer chain.

Introduction

Dynamics of granular media has been a subject of great research interest owing to its application in the field of condensed matter and solid state physics, soil dynamics, etc. These systems comprise of discrete particles (beads) and the dynamics is governed by the macroscopic geometrical shape and elastic properties of the interacting particles. The dynamics of one-dimensional granular chain of spherical beads has been very well studied theoretically, numerically and experimentally [1, 2]. The homogenous system supports solitary waves which are localized propagating disturbances that remain unattenuated [1, 2]. In contrast to homogeneous granular chains, uncompressed granular 1:1 dimer chain (ref. Fig. 1(a)) supports solitary wave propagation at discrete spectrum of mass ratios (ε) (ref. Table 1) between the light and the heavy beads [3].

It has been observed that the analytical study of the dynamics of granular media is extremely complex due to the truly nonlinear dynamics and bead separation in the absence of precompression. However, there are many studies which show that the simple Binary Collision Approximation (BCA) can provide an insight to such complex dynamics. The Binary Collision (BC) model considers mutual interaction of a maximum of two particles at any instant of time. Thus, the conservation of momentum and a relation for the coefficient of restitution is sufficient to compute the post-collision velocity of beads. But, equivalence between the BC model and the granular chain is seldom possible; however, they exhibit certain similarity in their wave propagation characteristics. For example, it is known that homogenous and dimer granular chains support solitary waves. Similarly, homogeneous BC model exhibits energy propagation localized on a single bead at any instant of time. The primary objective of the current work is to explore the spectrum of mass ratio that supports solitary wave propagation in dimer BC model. Further, we intend to find a qualitative and quantitative correspondence of the spectrum of mass ratio and the velocity response between the dimer BC model and the granular dimer [4].

Dynamics of Dimer BC Model and Solitary Waves

We denote the j^{th} heavy and light bead as H_j and L_j respectively (ref. Fig. 1(b)) which are initially at rest with equal inter bead distance d. The time of interaction of two beads is a function of the incoming velocities, mass ratio (ε) and the interaction potential. In contrast, the post-collision velocity of beads is a function of the incoming velocities and ε and is independent of the interaction potential. In this study we consider only the post-collision velocities of the beads and thus the time of interaction between beads is considered to be zero (i.e. the limit of hard spheres) such that it results in instantaneous change in bead velocity upon collision. Without loss of generality we consider $d = 5$ and unit impulse applied on the first bead (H_1) of the dimer BC model shown in Fig. 1(b). The bead H_1 collides with L_1 and transfers a part of its energy to L_1. The bead L_1 then collides with the H_2 and bounces back. We primarily concentrate on the collision event between $L_1 - H_2$ and denote it as the i^{th} rebound $(i \in \mathbb{Z}^+)$. For complete energy propagation and thereby realization of solitary waves in a lattice, the velocity of the beads in the trail of the primary pulse needs to be zero and the system should reach a stationary state after the passage of the wave as observed in homogeneous and at the discrete spectrum of mass ratios (ref. Table 1) in granular dimer chains. However, for arbitrary mass ratio, the velocity of the beads never decay to zero and the wave loses a fraction of its energy as it traverses through the periodic pairs of heavy-light beads. Thus, all the beads have finite velocity in the trail of primary pulse and seldom reach a stationary state. Such a behavior is observed even in the dimer BC model. Thus, the problem at hand is to determine the possible values of ε for which all the beads reach a stationary state in the trail of the primary pulse in the dimer BC model. Accordingly, we formulate the following three conditions considering beads H_j, L_j and H_{j+1}:

1. Bead H_j should reach a stationary state by the time L_j experiences i^{th} rebound, i.e. the collision between $H_j - L_j$ just before the i^{th} rebound (between L_j and H_{j+1}) should render H_j stationary.
2. Bead L_j should reach a stationary state immediately upon i^{th} rebound.

3. During this phase of $H_j - L_j - H_{j+1}$ interaction, H_{j+1} should be interacting only with L_j and cannot interact with L_{j+1}. If collision occurs between H_{j+1} and L_{j+1} during this phase, the wave splits and solitary wave cannot be realized. By imposing these conditions we effectively transfer all the energy to H_{j+1} after the i^{th} rebound and thereby drive both the beads H_j and L_j to a stationary state.

Based on the scheme described above, one can formulate the velocity of beads H_j, L_j and H_{j+1} after i^{th} rebound which are functions of mass ratio ε. Accordingly, one can deduce an unique value of $\varepsilon \leq 1$ corresponding to each value of i such that $H_j = 0$, $L_j = 0$ and $H_{j+1} = 1$ after i^{th} rebound. The spectrum of mass ratio thus deduced is presented in Table 1. The velocity response of the dimer BC model corresponding to $i = 3$ is presented in Fig. 2a and the corresponding fast frequency response of the light beads in granular dimer is presented in Figure 2b respectively. As can be observed, a qualitative match between the responses can be deduced by observing the peaks of these responses. Although, closed form expressions for the velocities of the beads after an arbitrary i^{th} rebound is seldom possible, one can conjecture that a spectrum of mass ratio that support solitary wave propagation in dimer BC model can be predicted close to $1/i^2$. The approximation is presented in Table 1 and a very good match with the dimer BC model can be clearly observed.

Conclusions

In this exposition we explore solitary wave propagation in dimer BC model and compare the spectrum of mass ratio and the dynamics with those supported by uncompressed granular dimer chains. Based on the idea of time scale separation in granular dimer chain, we formulate a simple BC model wherein the light bead interacts i times with its neighboring heavy beads before reaching a stationary state. With such a scheme a spectrum of mass ratios is analytically deduced which correspond to propagating solitary wave in the dimer BC model and numerically verified. A one-to-one correspondence between the spectrum of mass ratios supporting solitary waves in granular dimer and dimer BC model is observed. The responses of the light bead are found to be symmetric about a point of zero relative velocity of the neighboring heavy beads in both the dimer chains.

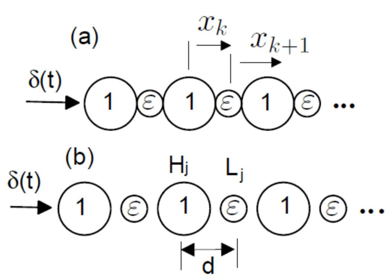
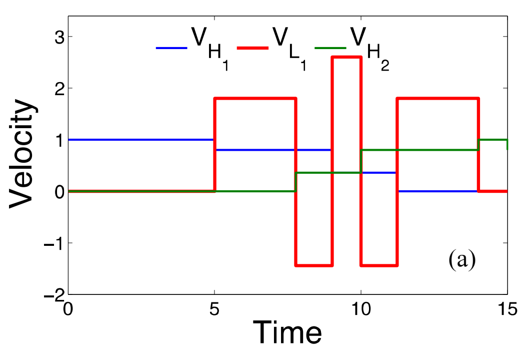


Figure 1: Schematic of a) Granular dimer and b) Dimer BC model

i	Dimer BC model ($\varepsilon_{sw}^{(i)}$)	Granular dimer ($\tilde{\varepsilon}_{sw}^{(i)}$)	$1/i^2$
1	1	1	1
2	0.236068	0.3428	0.25
3	0.109916	0.1548	0.1111
4	0.064173	0.0901	0.0625
5	0.0422171	0.0615	0.04
6	0.0299278	0.04537	0.0278
7	0.0223406	0.03448	0.0204

Table 1: Spectrum of mass ratios corresponding to dimer BC model and granular dimer



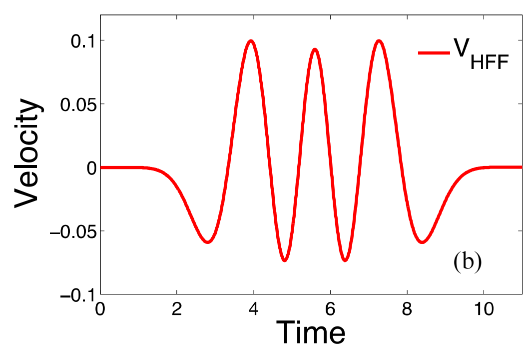


Figure 2: Velocity response of beads for a) dimer BC model ($\varepsilon_{sw}^{(3)} = 0.109916$) and b) velocity fast frequency component of a light bead in granular dimer chain ($\tilde{\varepsilon}_{sw}^{(3)} = 0.1548$).

References

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