Thermalization of a coupled oscillator chain

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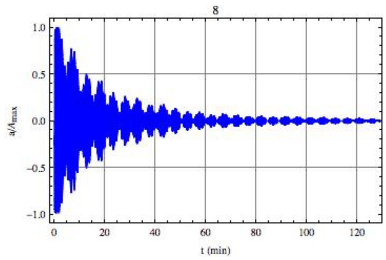
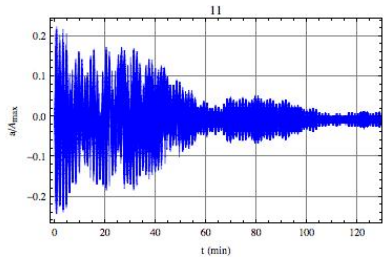
Summary. Coupled pendula show complex and unpredictable collective motions and provide a suitable physical model for complex dynamical systems. Starting from the well-known Fermi-Pasta-Ulam experiment coupled oscillators are expected to undergo spontaneous thermalization, typical of multi-body systems with non-linear interactions, and have been studied in order to investigate energy equipartition and second principle of thermodynamics. By means of an automated videotracking apparatus we have monitored both single and collective motions occurring in a chain of 24 non-linearly coupled pendula on varying the initial conditions (anharmonicity level, number and energy of excited pendula, etc.). Compared to the original FPU model our chain is highly and quickly dissipative and thermalizes very early. Moreover, we have observed other noticeable phenomena, e.g. some chaotic behaviour, spatial oscillation asymmetry, intrinsic localized modes.

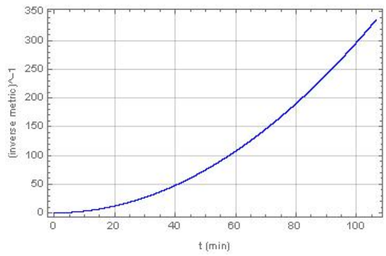
Some results

Thermalization

We studied each oscillator motion for the whole run. Observing the following space-time plot for the excited pendulum (8th) we can recognize pulses on a 5 minutes scale, but larger scale pulses (20 to 30 minutes) can be noticed in non excited pendulum plots (e.g. 11th). We calculated the energy inverse metric for the chain: it measures its termalization degree as a function of time.



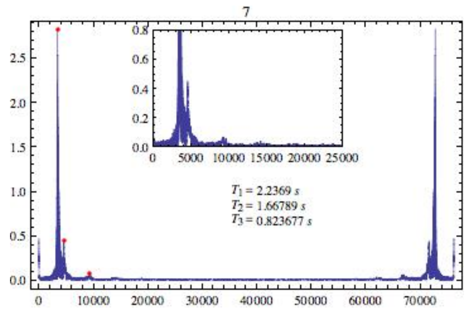




Fourier Analysis

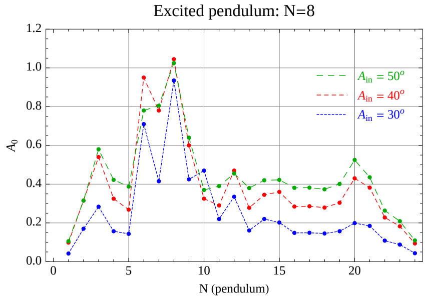
We obtained the Fourier Spectrum for each oscillator motion, as shown in the related plots, where oscillation period is provided (inverse of frequency). T_1 indicates the dominant period. The dominant period T_1 is approximately of 2.2 s, corresponding to $2\pi\sqrt{L/g}$. Few other pendula show two secondary peaks, probably related to first superior harmonics of the ground frequency. Hereafter we present the table of calculated periods and the Fourier spectrum for one pendulum (7th), where red dots identify the main peaks.

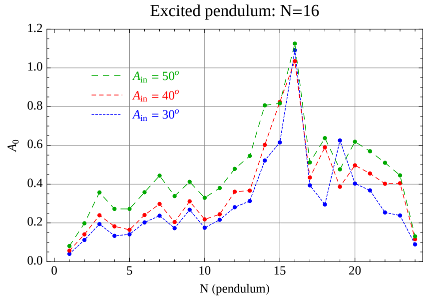
Pendolo	T_1	T_2	T_3
1	2.22448	1.71541	0.831693
2	2.22458	1.71738	0.860043
3	2.22461	1.72438	0.858216
4	2.22461	1.71778	0.862192
5	2.23111	1.71778	0.856578
6	2.21685	1.71778	0.843791
7	2.2369	1.66789	0.823677
8	2.18685	1.61722	1.06747
9	2.23756	1.66534	0.822611
10	2.2284	1.66498	0.834767
11	2.23559	1.70481	0.803795
12	2.22645	1.87776	0.896602
13	2.2938	X	X
14	2.22577	X	X
15	2.23931	X	X
16	2.23931	X	X
17	2.23931	X	X
18	2.23931	X	X
19	2.19568	X	X
20	2.19568	X	X
21	2.19568	X	X
22	2.19568	X	X
23	2.19568	X	X
24	2.19568	X	X



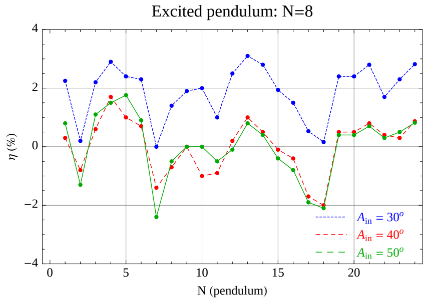
Horizontal Analysis

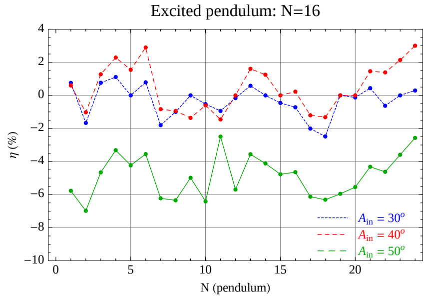
We performed 3 different runs, with the same excited pendulum (8th or 16th), but with different initial angles (30°, 40° and 50°). We compared the maximum amplitude for each pendulum in different runs. The results are shown in the following plots, where we can notice that the excited pendulum does not transfer most its energy to its next, rather to the ones next to that.



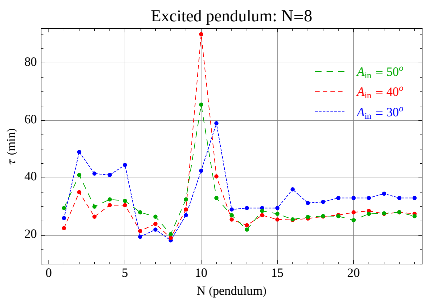


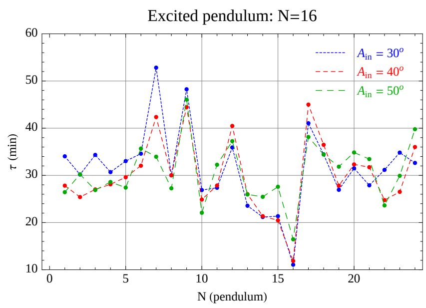
One of the main results we want to mention is related to asymmetry. The average position of each pendulum over the whole run tends not to correspond to its rest point. Individual pendula are oscillating around positions very far from the chain zero, as much in the positive direction as in the negative. It could be due to a coherence effect. This is an unexpected and original result of our research, because that phenomenon has not been reported in the literature. The oscillation asymmetry could be observed in collective resonance phenomena and in particular in the distributed coupled oscillations of buildings and mechanical machines.





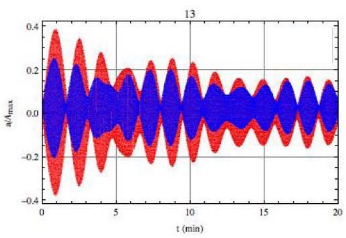
We also point out that pendula with higher energy get damped more quickly than the others and that the pendula at the end of the chain get less energy than the others. This phenomenon could be related to the nearby constraints. This is shown in the following plots where relaxation time τ (i.e. the time constant of the exponential decrease of the oscillation amplitude) of each pendulum in the chain is compared in the different runs. For most of the pendula τ is around 30 minutes, but it also assumes much smaller or larger values for specific pendula. Moreover we notice that small variation on the initial angle produce as a result that a local minimum becomes a local maximum or viceversa.

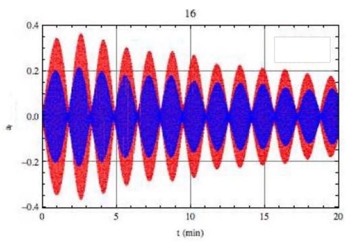




Chaos

In order to investigate the occurrence of chaos we compared two runs with same initial conditions and we overlapped the two plots of normalised amplitude as a function of time, as it is shown in the following figure for pendula 13th and 16th.





The large difference of the two plots shows that the system is sensitive to small variations to the initial conditions as it is expected for a chaotic system.

References

[1] T. Dauxois, M. Peyrard, S. Ruffo: *The Fermi-Pasta-Ulam "numerical experiment": history and pedagogical perspectives*, Eur.J.Phys. **26**, S3 (2005)

[2] G. Gallavotti (Ed.): *The Fermi-Pasta-Ulam Problem: A Status Report*, Lect.Notes Phys., **728** (Springer, Berlin Heidelberg 2008)