Analytical Solution for Energy Harvesting from Nonlinear Transverse Vibration of an Asymmetric Bimorph Piezoelectric Plate

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Summary. In this paper, analytical relation for harvesting of electrical power from nonlinear vibration of an asymmetric bimorph piezoelectric plate is presented based on classical plate theory and von Kármán strain-displacement relations in presence of temperature change effects. Two coupled ODEs for displacement and voltage are derived and solved via multiple scales, and the effects of some parameters are investigated.

Introduction

Vibration is usually known as a harmful phenomenon in mechanical systems and researchers have always been trying to prevent the system from that using damping mechanisms or absorbers. New researches are also focused on exploiting vibration, e.g. as a source for generating energy. This field is known as vibration energy harvesting.
Different materials and mechanisms are used to convert mechanical vibration into electrical power. One of these methods uses smart materials and in particular piezoelectric layers. Piezoelectric energy harvesting for a bimorph beam has been addressed in many studies [1, 2, 3]. In the present study, piezoelectric energy harvesting for bimorph plate is investigated analytically. The main contribution consists of deriving a closed form relation for harvesting the electrical voltage from nonlinear transverse vibration of an asymmetric plate with two different thickness of piezoelectric layers in top and bottom of the main plate, based on classical plate theory and nonlinear von-Kármán strain-displacement relation in presence of temperature change effects.

Coupled nonlinear equations and effects of parameters

Figure 1 shows the structure of the asymmetric piezoelectric plate with edges length *a* and *b*, with h_s thickness of the substructure and h_{p1} and h_{p2} thicknesses of the bottom and top piezoelectric, respectively. Constitutive equations for the *k*th piezoelectric layer are:

$$\begin{aligned}\{\sigma\}^k &= [Q]^k\{\varepsilon-\alpha\Delta T\}-[e]^k\{E\}\\ \{D\}^k &= [e]^{kT}\{\varepsilon-\alpha\Delta T\}+[\in]^k\{E\}\end{aligned} \tag{1}$$

where $\{\sigma\}$, $\{E\}$, $\{\varepsilon\}$, $\{D\}$ and $\{\alpha\}$ are the vectors of the stress, electric field, strain and electric displacement and thermal expansion factor, respectively. Also ΔT is the operation-induced temperature change. $[Q]$,$[e]$ and $[\in]$ are the matrices of elastic stiffness, piezoelectric coefficients and permeability coefficients. Based on classical plate theory, the governing equations of motion under distributed transverse forcing are:

$$\begin{aligned}&\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\ &\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=0\\ &\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+\frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x}+N_{xy}\frac{\partial w_0}{\partial y}\right)+\frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w_0}{\partial x}+N_{yy}\frac{\partial w_0}{\partial y}\right)+q(x,y,t)=I_0\frac{\partial^2 w_0}{\partial t^2}-I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)\end{aligned} \tag{2}$$

Figure 2 shows the equivalent circuit of the piezoelectric plate at parallel state. Using equations (1), considering von-Karman relations, and applying Galerkin method, a coupled nonlinear ODE for transverse vibration of plate is derived as:

$$\ddot{W}(t)+\omega^2W(t)+2\mu'\dot{W}(t)+s_1W^2(t)+s_2W^3(t)+s_3W(t)V(t)+s_4q(t)=0 \tag{3}$$

where ω, μ' are natural linear frequency and damping coefficient respectively, s_i are constant coefficients including the mechanical and electrical properties of main and layer structures, and *W(t)*, *V(t)* and *q(t)* are transverse displacement, output voltage of the structure and excitation force, respectively.

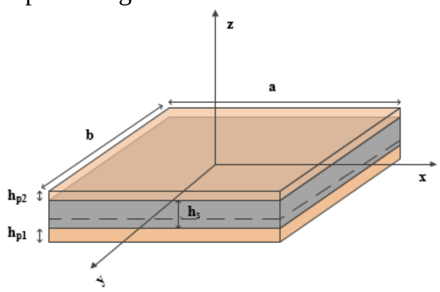

Figure 1: Asymmetric piezoelectric plate

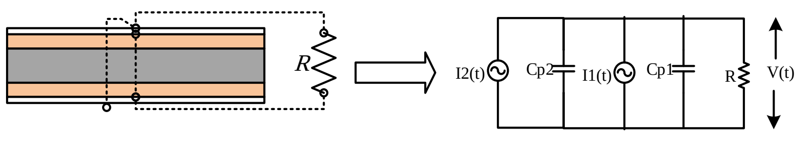

Figure 2: Equivalent circuit of the structure.

Electric equation is obtained by electric displacement definition as [3]:

$$\frac{d}{dt}\left(\int_A D_z.\mathrm{n}\,dA\right)dt = \frac{V}{R} \tag{4}$$

where D_z is electric displacement in thickness direction, A is plate surface and R is load resistance. Eq (5) is obtained after integrating eq (4) as:

$$\dot{V}(t) + J_1 V(t) + J_2 \dot{W}(t) + J_3 W(t)\dot{W}(t) = 0 \tag{5}$$

where Js are constant coefficients. The nonlinear equations (3) and (5) are solved by multiple time scale method considering primary resonance, and finally an analytical relation for the electric voltage of energy harvesting is obtained for the first time as:

$$V(t) = \frac{J_2 K\omega}{J_1^2+\omega^2}\left(J_1 \sin(\beta+\omega t) - \omega\cos(\beta+\omega t)\right) + \frac{J_3 K\omega}{J_1^2+4\omega^2}\left(\frac{J_1}{2}\sin(2\beta+2\omega t) - \omega\cos(2\beta+2\omega t)\right) \tag{6}$$

where K and β are vibration amplitude and frequency of external harmonic excitation, respectively. The power is calculated in one linear period as:

$$Power = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{V^2}{R}\,dt \tag{7}$$

Diagrams of Power versus detuning parameter, load resistance and excitation amplitude are shown in figures 3, 4 and 5 respectively for a specific problem. The frequency-response curve is shown in figure 6. It is found that the power has a direct relation with detuning parameter and excitation amplitude and a reverse relation with load resistance. It is also found that damping coefficient has a minor effect on the power.

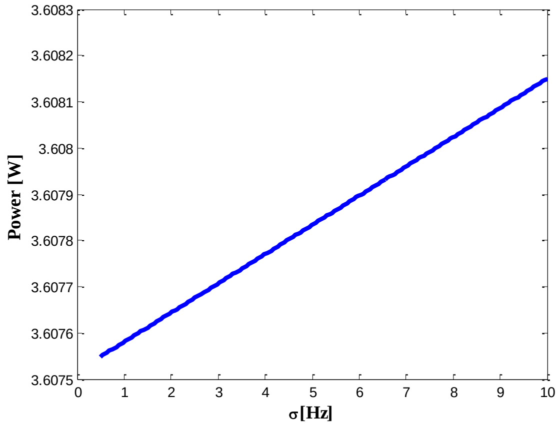

Figure (3): Diagram of power versus detuning parameter

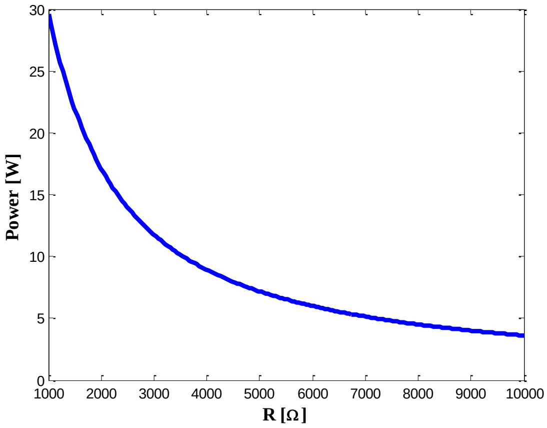

Figure (4): Diagram of power against lod resistance

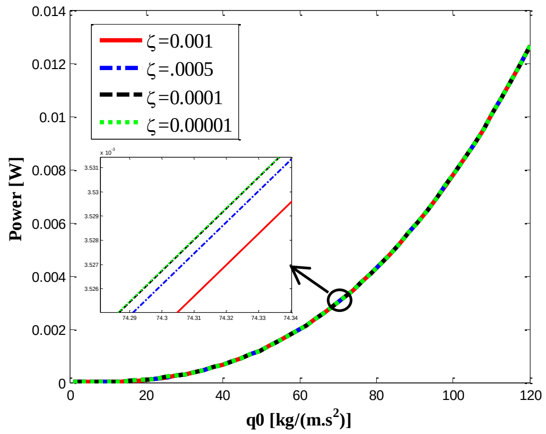

Figure (5): Diagram of power against excitation amplitude

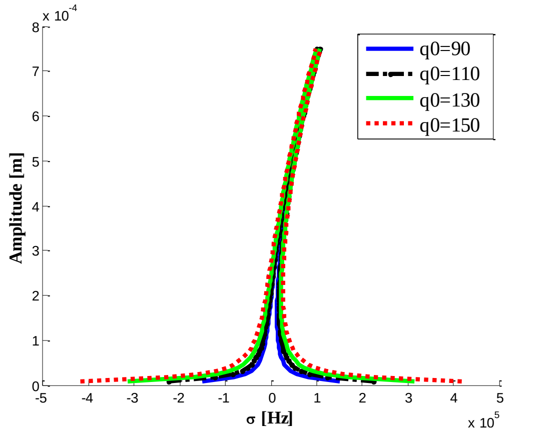

Figure (6): Diagram of amplitude response against detuning parameter

References

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