NONLINEAR VIBRATIONS OF FUNCTIONALLY GRADED SHALLOW SHELLS OF A COMPLEX PLANFORM IN THERMAL ENVIRONMENTS

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Summary. Geometrically nonlinear vibrations of FGM shallow shells of an arbitrary planform subjected to thermal environment are investigated with the use of R-functions theory and variational methods. Nonlinear first-order shear deformation shallow shells are employed. Material properties are assumed to be temperature-dependent and varying along the thickness direction accordingly to Voigt's law. The developed method is based on the combined applications of R-functions theory, variational Ritz's method, procedure by Bubnov-Galerkin, and Runge-Kutta's approach. The effect of the temperature rise, geometry of the shell, and constituent volume fraction index is examined. A comparison of the obtained results with available ones is also carried out for rectangular plates and shallow shells.

Keywords R–function theory, FGM shallow shells, geometrically nonlinear vibrations, thermal environment

Introduction

Functionally Graded Materials (FGM) are among the most effective materials that can be used in high temperature environments. Initially, FGMs were designed as thermal barrier materials for nuclear reactors, space planes, but now, they are widely used in different industries such as fast computer, biomedical industry, chemical plants, and of course aerospace. A large number of publications devoted to the study of FGM plates and shells as the main elements of many modern designs appeared in the last two decades. Review of the main achievements in this area can be found in many publications [1-4]. An analysis of available publications has showed that there are practically no studies of plates and shells of a complex geometric form. This problem can be investigated by applying the effective numerical-analytical approach based on the R-functions theory and variational methods (RFM).

The problem formulation

Assume that a shallow shell is made from a mixture of ceramic (top of the shell) and metal (bottom). The dependency of the temperatures of constituent materials is taken into account using the following formula [5]

$$P(T) = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right)$$

The effective material properties *P(T,z)* such as Young's modulus *E*, Poisson's ratio ν , mass density ρ , and coefficient of the thermal expansion α can be expressed as [6]

$$P(T,z) = \left(P_c(T)V_c(z) + P_m(T)V_m(z)\right),$$

where V_c, V_m are ceramic and metal volume fractions. They are related as $V_c + V_m = 1$. Below a power-law distributions (Voigt's model) of the volume fractions of the metal and ceramic is employed

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k, \quad V_m(z) = 1 - \left(\frac{z}{h} + \frac{1}{2}\right)^k .$$

Assume that the temperature is varied only in the thickness direction. In this case, the temperature field is modeled by one-dimensional heat transfer equation of the following type:

$-\dfrac{d}{dz}\left(k(z)\dfrac{dT}{dz}\right) = 0$, where $T = T_m$ at $z = -h/2$ and $T = T_c$ at $z = +h/2$. As shown in [7], the equation may be solved by employing the polynomial series. The mathematical statement of the nonlinear vibrations problem is fulfilled in the framework of the FSDT of the shallow shells. The membrane stress resultants $\{N\}$ and bending stress resultants $\{M\}$ are presented as [1,2]: $\{N\} = \{N_{xx}, N_{yy}, N_{xy}\}^T = [A_{ij}]\{\varepsilon\} + [B_{ij}]\{\chi\} - \{N^{th}\}$
$\{M\} = \{M_{xx}, M_{yy}, M_{xy}\}^T = [B_{ij}]\{\varepsilon\} + [D_{ij}]\{\chi\} - \{M^{th}\}$ where the matrices $[A_{ij}],[B_{ij}],[D_{ij}]$ $i, j = (1,2,6)$ are defined as

$\left[A_{ij}, B_{ij}, D_{ij}\right] = \int_{-h/2}^{h/2} \left[Q_{ij}\right]\left(1, z, z^2\right) dz$, and $Q_{11} = Q_{22} = \frac{E(T,z)}{1-\nu^2}, \quad Q_{12} = \nu Q_{11}, \quad Q_{16} = Q_{66} = 0, \quad Q_{44} = Q_{55} = Q_{66} = \frac{E(T,z)}{2(1+\nu)}$. The thermal resultant $\left\{N^{th}\right\}$and moment resultant $\left\{M^{th}\right\}$are described by the following formulas:

$$\left\{N^{th}\right\} = \left\{N_{xx}^{th}, N_{yy}^{th} N_{xy}^{th}\right\}^T = \int_{-h/2}^{h/2} \left[Q_{ij}\right]\left\{\alpha(T,z,), \alpha(T,z,), 0\right\}^T \Delta T(z) dz, \quad \left\{M^{th}\right\} = \left\{M_{xx}^{th}, M_{yy}^{th} M_{xy}^{th}\right\}^T = \int_{-h/2}^{h/2} \left[Q_{ij}\right]\left\{\alpha(T,z,), \alpha(T,z,), 0\right\}^T z \Delta T(z) dz$$

The transverse shear force $\{QP\} = \left\{QP_{xz}, QP_{yz}\right\}$ is also related to the transverse shear strain $\{\varepsilon_{13}, \varepsilon_{23}\}$ by expression:

$\{QP\} = [C_{ij}]\{\varepsilon_s\}, \quad \left[C_{ij}\right] = \int_{-h/2}^{h/2} \left[Q_{ij}\right] k_i k_j dz, \quad (i, j = 4{,}5)$, where k_i is the transverse shear coefficient.

Method of solution

The proposed method consists of a few steps. First, matrices $\left[A_{ij}\right], \left[B_{ij}\right], \left[D_{ij}\right] (i, j, = 1{,}2{,}6), \left[C_{ij}\right] (i, j, = 4{,}5)$ and values $\{I_0, I_1, I_2\} = \int_{-h/2}^{h/2} \rho(T,z)\left(1, z, z^2\right) dz$ are calculated. In the second step, the linear analysis is conducted to find natural frequencies and corresponding natural modes to be applied while solving nonlinear problem. In the next step, sequences of auxiliary problems, like elasticity problems, are solved. The natural modes and solutions of auxiliary elasticity problems are used in the fourth step as a basis for expanding the nonlinear displacements. Further, the Bubnov-Galerkin procedure is applied to reduce initial equations of motion to a system of ordinary differential equations (ODEs). Finally, the obtained ODEs are solved by the Runge-Kutta method.

Numerical Results

The validation and accuracy of the proposed method were verified on a large number of the test problems. As an example, let us consider a comparison of the fundamental frequency with known results [3] for simply –supported square FG plate, as shown in Table 1. To check the accuracy of results obtained for geometrically nonlinear vibration of the spherical simply-supported panel made of two different mixtures, two cases are studied: a) mixture of silicon nitride (Si_3N_4) and stainless steel $(SUS304)$ and b) $ZrO_2 / Ti-6Al-4V$. The comparison of results obtained for the power-law index *k=2* is presented in Fig.1. The proposed method is demonstrated for FG clamped and simply-supported shallow shells of a complex planform also. The effect of the power-law index, properties of the constituent materials, curvature of the shell, boundary conditions, and geometry of the shell at different values of temperature at the top and bottom surfaces are analyzed.

Table 1. Comparison of the fundamental linear frequency parameters $\Omega_L = \omega_L (a^2/h)\left(\rho_m\left(1-\nu^2\right)/E_m\right)^{1/2}$ of the simply supported FG ($Si_3N_4/SUS304$) of square plate (*a/b=1,h/a=0.125*).

T_m, T_c	Method	$k=0$	$k=0.5$	$k=1$	$k=2$	$k=100$
300K, 300K	RFM	12.532	8.624	7.553	6.788	5.421
	[3]	12.495	8.675	7.555	6.777	5.405
300K, 400K	RFM	12.209	8.409	7.367	6.620	5.351
	[3]	12.397	8.615	7.474	6.693	5.311
300K, 600K	RFM	11.625	7.998	7.012	6.310	5.043
	[3]	11.984	8.269	7.171	6.398	4.971

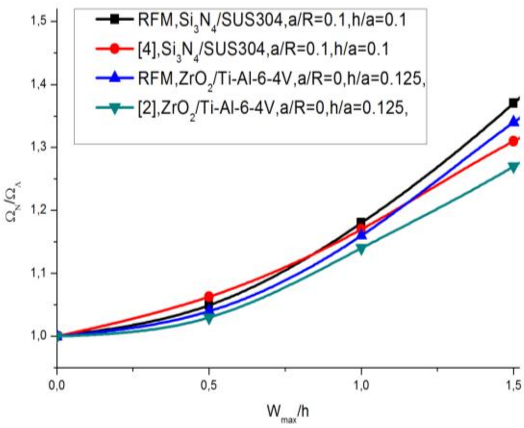

Fig.1

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