Experiment and Analysis of Active Vibration Suppression via an Absorber with a Tunable Delay

Yixia Sun [*] and Jian Xu [**]

[*]*School of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai, China*

[**]*School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai, China*

Summary. A time-delayed absorber is presented to suppress the vibration of a primary system when the passive absorber loses efficiency. Both the inherent and the intentional time delays are considered in the feedback loop. The effects of the feedback gains and intentional time delay on vibration suppression are studied experimentally and theoretically. The experimental results show that reasonable feedback gain and intentional time delay may improve the absorber's vibration suppression effectiveness.

Introduction

Time-delayed absorber is a novel vibration control technique. The key idea of a time-delayed absorber is the introduction of an actuator controlled via state feedback with the adjustable time delay and feedback gain. Olgac [1] designed a delayed resonator (DR) to suppress the vibration of a harmonically forced system. With proper choices of feedback gain and time delay, DR rests the controlled system at the desired frequency. Tootoonchi [2] used DR to minimize the relative vibration between the cutting tool and work piece. Zhao [3] investigated the effect of positive and negative feedback on the performance of a nonlinear time-delayed absorber. El-Sayed [4] observed the effect of a time-delayed absorber on suppressing the vibration due to rotor blade flapping motion. Although, there have been extensive theoretical studies about the time-delayed absorber, few efforts have been devoted to experiments.
The vibration control effect of the time-delayed absorber with inherent time delay has been studied experimentally in our previous work [5]. It is found that the proper utilization of inherent time delay greatly improve the performance of vibration suppression. Motivated by this finding, an intentional time delay is introduced in the controller, the combined effects of inherent and intentional time delays are studied experimentally in this paper.

Modeling and stability analysis

Fig. 1 illustrates the mechanical model of the 2-dof combined system. The system consists of a time-delayed absorber and a primary system. m_1 and m_2 represent the mass of the absorber and the primary system, respectively. x_1 and x_2 denote the displacement of the absorber mass and the primary mass. An actuator provides the feedback control force. Assuming that a harmonic excitation is applied to the primary system, the governing equations of the combined system are given by

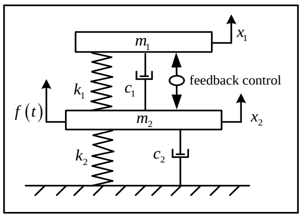

Fig. 1 Mechanical model

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) - k_c x_c = 0 \tag{1}$$

$$m_2\ddot{x}_2 + k_2 x_2 + c_2\dot{x}_2 + k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1) + k_c x_c = f\sin(\omega t) \tag{2}$$

where $k_c x_c$ is the time-delayed feedback control force and x_c is the control signal with the following form

$$x_c = \alpha_1\left[x_1(t-\tau_1) - x_2(t-\tau_1)\right] + \alpha_2\left[x_1(t-\tau_1-\tau_2) - x_1(t-\tau_1)\right] \tag{3}$$

where τ_1 and τ_2 represent the inherent and the intentional time delay in the feedback control loop, respectively. α_1 and α_2 indicate the amplification factors of the control signal. Substituting Eq. (3) into Eqs. (1) and (2) yields

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) - g_1 x_1(t-\tau_1) + g_1 x_2(t-\tau_1) - g_2 x_1(t-\tau_1-\tau_2) + g_2 x_1(t-\tau_1) = 0 \tag{4}$$

$$m_2\ddot{x}_2 + k_2 x_2 + c_2\dot{x}_2 + k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1) + g_1 x_1(t-\tau_1) - g_1 x_2(t-\tau_1) + g_2 x_1(t-\tau_1-\tau_2) - g_2 x_1(t-\tau_1) = f\sin(\omega t) \tag{5}$$

where $g_1 = \alpha_1 k_c$ and $g_2 = \alpha_2 k_c$ are feedback gains. The solutions of Eqs. (4) and (5) can be expressed as

$$x_1 = a_1\sin(\omega t) + b_1\cos(\omega t), \quad x_2 = a_2\sin(\omega t) + b_2\cos(\omega t) \tag{6}$$

By substituting Eq. (6) into Eqs. (4)-(5) and extracting the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$ from both sides of the equations, we obtain the linear algebraic equations about a_1, b_1, a_2 and b_2: $\{a_1, b_1, a_2, b_2\}^T = [A]^{-1}[F]$ (7)

Hereinafter, we define H_1 and H_2 as the acceleration transfer functions of the absorber and the primary system:

$$H_i = \frac{\|\ddot{x}_i\|}{f} = \frac{\omega^2\sqrt{a_i^{\,2} + b_i^{\,2}}}{f}, \quad (i = 1, 2) \tag{8}$$

Through the above derivation, the period responses of the combined system are obtained. It is known that feedback gain and time delays have a great influence on the stability of the system responses. Hence, it is necessary to analysis the stability of the system responses. The characteristic equation of the combined system is

$$P_0(s)+P_1(s)e^{-\tau_1 s}+P_2(s)e^{-(\tau_1+\tau_2)s}=0 \tag{9}$$

The combined system is stable if and only if all characteristic roots of Eq. (9) have negative real parts.

Experimental results

The schematic view of the time-delayed feedback control experiment is illustrated in Fig. 2. A shaker (5) and a power amplifier act as the excitation source and provide the sinusoidal excitation force to the primary mass (2). The frequency of the excitation force is set in M+p vib-pilot, which works as a signal generator. Force sensor (6) and acceleration sensors (7), (8) are respectively used to monitor the excitation force and the responses of absorber mass (1) and primary mass. The feedback control loop is described as follows. Firstly, the acceleration signals of sensors (7) and (8) enter into signal conditioning instrument, in which the acceleration signals are amplified and filtered to improve signal-to-noise ratio. Secondly, the processed signals go into trio motion controller via voltage lifting device. The time-delayed feedback control command (i.e. Eq. (3)) is written in trio motion controller. Thirdly, the control command is transferred into servo controller which guides the rotation of the shaft of servo motor (3). Finally, the lower end of controlled steel sheet (4) moves horizontally under the drive of servo motor shaft and applies the feedback control force.

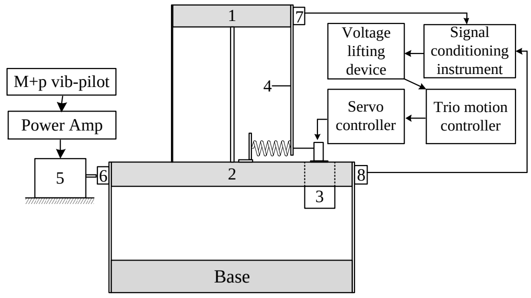

Fig. 2 Schematic of time-delayed feedback control

The physical parameters of the combined system are: m_1 =0.667kg, m_2 =6.8kg, k_1 =2431.72N/m, k_2 =13502.4N/m, k_c =198.96N/m, c_1 =2.9Ns/m, c_2 =4.1Ns/m and τ_1 =63ms [5]. Hereinafter, $\Omega=\omega/2\pi$ (in hertz) denotes excitation frequency.

Fig. 3 shows the variations of H_2 versus α_2 for $\Omega=10.25\text{Hz}$, $\alpha_1=0.4$ and τ_2=30ms . The line represents the analytical results determined according to Eq. (8). The red dots represent experimental results. It's obvious that the time-delayed absorber minimizes the vibration of the primary system for $\alpha_2=-0.7$. To observe the effect of the intentional time delay on the vibration suppression effect, Fig. 4 shows how H_2 change as a function of τ_2 when $\Omega=10.25\text{Hz}$, α_1=0.8 and α_2=0.5 . It is found that the value of H_2 fluctuates with the increase of τ_2 . There is a peak value when $\tau_2\approx 40\text{ms}$ and a valley value when $\tau_2\approx 85\text{ms}$. Fig. 5 shows the measured time histories of the excitation force and the corresponding system responses for $\Omega=10.25\text{Hz}$. It is seen that the value of H_2 decreases 71.93% when the time-delayed feedback is applied.

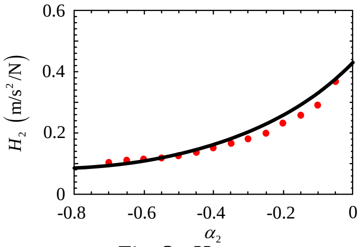

Fig. 3 H_2 versus α_2

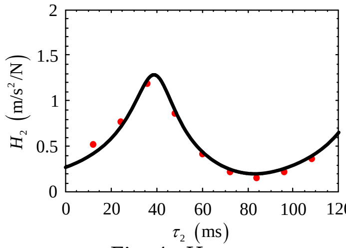

Fig. 4 H_2 versus τ_2

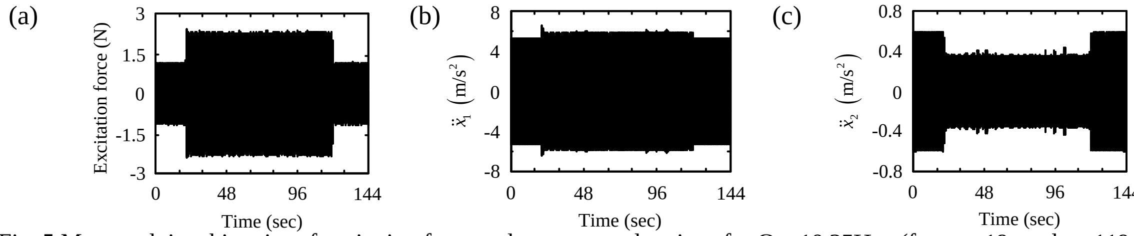

Fig. 5 Measured time histories of excitation force and system accelerations for $\Omega=10.25\text{Hz}$. (for $t<19\text{s}$ and $t>119\text{s}$, $\alpha_1=\alpha_2=0$; for $19\text{s}\le t\le 119\text{s}$, $\alpha_1=0.8$, $\alpha_2=0.5$ and τ_2=84ms), (a) excitation force, (b) absorber, (c) primary system

Conclusions

The effectiveness of a time-delayed absorber in suppressing the vibration of a harmonically excited primary system is demonstrated. An experimental device is designed to implement the time-delayed feedback control. Experimental and theoretical results point that the time-delayed absorber with proper choices of control parameters is effective for vibration suppression. It is noticed that for fixed feedback gains, sharp vibration suppression may occur at some values of intentional time delay. However, the time-delayed absorber fails at other values of intentional time delay.

References

[1] Olgac N., Holm-Hansen B. T. (1994) A Novel Active Vibration Absorption Technique: Delayed Resonator, *J. Sound Vib.*, 176: 93-104.
[2] Tootoonchi A. A., Gholami M. S. (2011) Application of Time Delay Resonator to Machine Tools, *Int. J. Adv. Manuf. Technol.*, 56: 879-891.
[3] Zhao Y.Y, Xu J. (2007) Effects of Delayed Feedback Control on Nonlinear Vibration Absorber System, *J. Sound Vib.*, 308: 212-230.
[4] El-Sayed, A. T., Bauomy, H. S. (2014) Vibration Control of Helicopter Blade Flapping via Time-Delay Absorber, *Meccanica*, 49: 587-600.
[5] Sun Y. X., Xu J. (2015) Experiments and Analysis for a Controlled Mechanical Absorber Considering Delay Effect, *J. Sound Vib.*, 339: 25-37.